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PRINCIPLES OF MECHANISM

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PREFACE

THIS book is intended to present the elementary principles of mechanism in a way that will make it adapted for use in evening technical schools, trade schools, mechanic arts high schools, and other schools where it is desired to teach the subject thoroughly yet without going into the highly mathematical treatment. Typical problems are solved throughout the text and a large number of problems are included for solution by the student.

W. H. J.
M. C. M.

CAMBRIDGE, 1917.

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PRINCIPLES OF MECHANISM

CHAPTER I

GENERAL DEFINITIONS

1. A Machine. A machine consists of a number of pieces or groups of pieces so arranged that, when a driving force is applied to the proper place, the various pieces operate together to do some useful work. Each of the pieces in a machine either moves or helps to guide some of the other pieces in their motion. A familiar example is the common sewing machine. Power is applied at the treadle, causing motion of the rod connected with the treadle; this motion is passed on up through the various parts with the final result that the cloth is stitched. It is evident that the machine consists of a frame and moving pieces; the frame supporting and guiding the moving pieces, while the moving pieces pass the forces along.

2. A Mechanism. A mechanism is one of the groups of pieces in a machine, all of which pieces are so connected that, when a definite motion is given to one of them, the others are caused to move in definite ways. The treadle, driving rod, crank shaft, and that part of the frame which supports them in the sewing machine, constitute a mechanism. A machine, therefore, consists of a series of mechanisms, each of which is doing its own work, and all of which work together to accomplish the purpose for which the machine is intended.

3. The Study of Mechanism. The study of mechanism is the study of the laws which govern the motions of the various parts of a machine and the forces which exist in or are

transmitted by those parts. That branch of the subject which deals with the motions only, regardless of the forces, is often given the name *Kinematics of Machines,* while the part dealing with forces is called *Dynamics of Machines.*

In this book only the kinematics of machines and a few of the simpler problems relating to forces will be treated.

4. Importance of a Study of Mechanism. The importance of this study to one who has to do with machinery is very great. Whether he is engaged in using or in designing machinery he ought to know how to analyze the motions which the various pieces have and to determine the speeds of the pieces. He also ought to know how to design and connect the mechanisms in order to obtain a desired motion for any particular piece. Furthermore, he needs to be able to determine the forces which may be expected to exist in the several pieces when a known force is applied at a known point. In short, in order intelligently to design or handle a machine, a man should thoroughly understand the natural laws which enter into the operation of the machine, and the various methods which have been devised to apply those laws to the performance of definite work. It is to this end that the science of mechanism is devoted.

5. Design. The design of a machine consists in adapting known mechanical appliances to meet special conditions. That is, there are certain elementary mechanical units such as levers, revolving wheels, screws, cams, cranks, connecting rods, sliders, etc., which form the basis of all mechanisms. The designer must so proportion and arrange these as to accomplish the desired end. It is purposed in the present work to enumerate some of the more common of these units or elements and to consider each one separately, discussing the natural laws governing its design and operation. Some of the ways in which these units are combined into mechanisms will be studied, also the motions and forces in these mechanisms. Sometimes it is better to carry on such studies by means of graphical work on the drawing board

and sometimes by calculation. Whichever method seems best in any particular case will be adopted in the present text, and very frequently both will be used as a check on each other.

Before beginning the consideration of a subject in which the terms *motion, velocity, speed, force,* and a number of other similar words frequently occur, it is desirable to get clearly in mind the exact meaning of these terms. The following definitions will explain briefly the sense in which the words are used in this discussion.

6. Motion. Motion is change of position. One can tell when an object is in motion only by comparing its position with that of some other object. In Fig. 1 if the cylinder

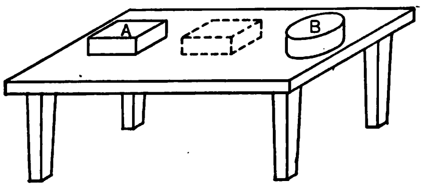


FIG. 1.

B is on the table and the block *A* is moving so that at the end of a minute it is in the position shown by the dotted lines, one-half inch nearer *B* than when it started to move, it can be said that *A* is moving relative to *B* at the rate of one-half inch per minute. If *B* is not moving, it can be said that the *absolute motion* of *A* is at the rate of one-half inch per minute. If *A* and *B* are both moving but there is enough difference in their motion so that *A* still approaches *B* at the same rate as before, we can still say that *the motion of A relative to B* is one-half inch per minute, although the actual or absolute motion of *A* would no longer be at the rate of one-half inch per minute. The motion of a body may, therefore, be described with relation to another moving body or to a body which is at rest. In either case its motion with respect to

the other body is its **relative motion.** Its motion with respect to a body which is not moving is its **absolute motion.**

7. Speed. Speed is rate of motion. It is the *number of units of space moved over in one unit of time.* For example, if a ball rolls along the ground a distance of 10 ft. in one minute, its speed is 10 ft. per minute. Speed may be uniform or variable. A body which moves with **uniform speed** moves over the same distance in each unit of time; that is, if it moves 1 in. the first second, 1 in. the second second, and so on, its speed is uniform. If, however, it moves 1 in. the first second, $1\frac{1}{2}$ in. the second second, 2 in. the third second and so on, its speed is said to be **variable.**

8. Velocity. Velocity is a word often used to mean the same as speed, although accurately speaking velocity includes not only speed but *direction.*

9. Acceleration. Acceleration is *rate of change of speed.* If for example a body is moving at the rate of 1 ft. per second at the end of the first second, 3 ft. per second at the end of the second second, 5 ft. per second at the end of the third second, etc., the speed is increasing at the rate of 2 feet per second each second. Its acceleration is two feet per second each second. Acceleration may be either positive or negative. If the speed is increasing the acceleration is **positive;** if the speed is decreasing the acceleration is **negative.** If the speed changes the same amount each second the acceleration is **uniform,** but if the speed changes by different amounts at different times the acceleration is **variable.**

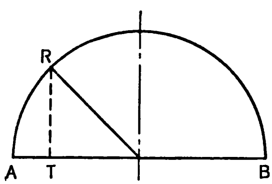


FIG. 2.

10. Uniform Motion. A body is said to have a uniform motion when its speed is uniform.

11. Harmonic Motion. If a point as R, Fig. 2, moves with uniform speed around the circumference of a semicircle

and another point T moves across the diameter in the same length of time, the speed of T varying so that it will always be at the foot of a perpendicular let fall from R to the line AB, then T is said to have harmonic motion.

12. Gravity Motion. If a body starts from rest and increases its speed by a uniform amount per unit of time until it reaches the middle point of the path over which it is to travel, then decreases its speed at the same rate that it increased it, during the last half of its motion, it is said to have "gravity" motion. This kind of motion is so named because a body falling freely by its own weight increases its speed by equal amounts each second, and a body thrown straight up into the air decreases its speed by equal amounts each second until it finally stops going up.

13. The Mechanical Elements. The ordinary mechanical units or elements may be grouped as follows:

- I. Stationary pieces
- II. Pieces revolving or oscillating about a fixed axis
  - Cylinders
    - Plain cylinders
    - Gears
    - Worms and worm wheels
    - Pulleys
  - Discs or flat plates
  - Cones
  - Spheres
  - Cranks
  - Levers
  - Cams
- III. Pieces having linear motion
  - Connecting bands
    - Belts
    - Ropes
    - Chains
  - Sliders
    - Moving along a straight path
    - Moving along a curved path
- IV. Pieces having motion consisting of rotation or oscillation about an axis which is itself moving
  - Connecting links, such as the connecting rod of an engine
  - Wheel rolling on the ground or other surface

V. Pieces which rotate about an axis while they move along that axis	Screws and nuts

The above tabulations cover practically the things which will be taken up in the following chapters, although the order may not be the same.

CHAPTER II

REVOLVING AND OSCILLATING BODIES

14. Revolving Wheel or Cylinder. One of the simplest and most common mechanical units is a wheel (cylinder) revolving on an axle or shaft. The shaft may turn with the wheel, being supported in bearings and kept from moving endwise by collars, or the shaft may be held stationary and the wheel turn on the shaft. An example of the first case is shown in Fig. 3 and of the second case in Fig. 4.

15. Angular Speed or Rate of Turning. Suppose some force is applied to the shaft in Fig. 3 so that it is caused to turn around, say, 75 times in a minute. If the wheel is fast to the shaft, the wheel will turn around 75 times in a minute. We would say, then, that the wheel makes 75 *revolutions per minute* (usually abbreviated thus: 75 r.p.m.) It is common practice to speak of the angular speed or speed of turning as so many revolutions per unit of time, usually per minute or second.

Another unit sometimes used for measuring angular speed is the angle called the *radian.* This is the angle which is subtended by the arc of a circle equal in length to the radius. Since the radius is contained in the circumference of a circle 2π times, there must be 2π radians in 360°, or one radian is equal to $360° \div 2 \times 3.1416 = 57° \ 17' \ 42''$.

If a revolving wheel turns once per minute, its angular speed as we have already seen is 1 r.p.m., and since one revolution of the wheel causes any radial line on the wheel to sweep over 360° or 2π radians, we can say that the angular speed of the wheel is 2π radians per minute. Now, if the

wheel turns N times per minute the angular speed is N r.p.m. or $2\pi N$ radians per minute. That is,

Angular speed in radians $= 2\pi \times$ number of revolutions. (1)

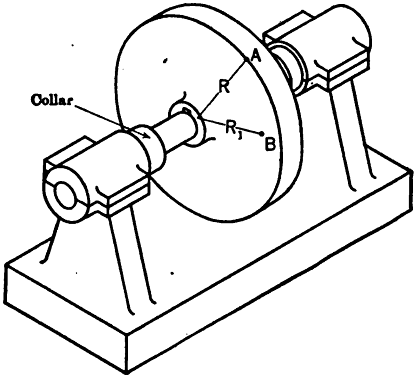


FIG. 3.

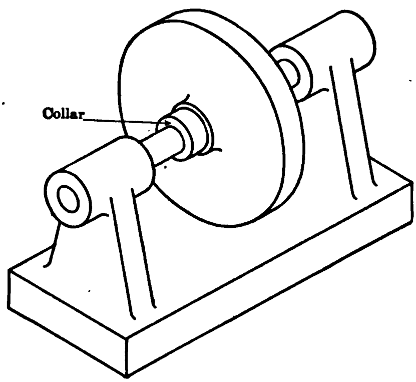


FIG. 4.

Example. If a wheel turns 90 r.p.m. its angular speed in radians is

$$2\pi 90 = 565.5 \text{ radians per minute.}$$

16. Linear Speed of a Point on a Revolving Body. Consider a point A on the circumference of the wheel in Fig. 3. While the wheel turns once, A travels over the circumference of a circle of the same diameter as the wheel, or it travels $2\pi R$ in. if the radius of the wheel is R in. Then, if the wheel turns N times in a unit of time, A travels over the circle N times in a unit of time. Therefore, the linear speed of the point A is $2\pi RN$ in. per unit of time, where R is the distance in inches of A from the center about which the wheel is turning and N is the number of times the wheel turns in a unit of time. Writing this out in the form of an equation we have,

$$\text{Linear speed of } A = 2\pi RN. \quad . \quad . \quad . \quad . \quad (2)$$

Example. Suppose the wheel is 12 in. in diameter and turns 40 times per minute. The speed of A would be $2\pi 6 \times 40$ in. per minute $= 1506$ in. per minute.

Now, from Eq. (1) we see that the angular speed is equal to $2\pi N$ radians; so that, dividing Eq. (2) by Eq. (1) we get,

$$\frac{\text{Linear speed of } A}{\text{Angular speed of wheel in radians}} = \frac{2\pi RN}{2\pi N} = \frac{R}{1},$$

or,

Linear speed of a point on a revolving body = angular speed of body in radians × distance of the point from the center. (3)

17. Relative Speed of Points at Different Distances from Axis. If another point B is chosen on the side of the wheel at a distance R_1 from the center, it can be shown in the same way that Eq. (2) was derived that,

$$\text{Linear speed of } B = 2\pi R_1 N. \quad . \quad . \quad . \quad . \quad (4)$$

Now, dividing Eq. (2) by Eq. (4),

$$\frac{\text{Linear speed of } A}{\text{Linear speed of } B} = \frac{2\pi RN}{2\pi R_1 N}.$$

Therefore,

$$\frac{\text{Linear speed of } A}{\text{Linear speed of } B}=\frac{R}{R_1}. \quad . \quad . \quad . \quad . \quad (5)$$

Thus, we see that the *linear speeds of two points on a revolving wheel are directly proportional to the distances of the points from the center about which the wheel is turning.*

The linear speed of a point on the circumference of a wheel is often spoken of as the *periphery speed* or *surface speed* of the wheel.

Let us take another case, that of two wheels fast to the same shaft as shown in Fig. 5. The weight P is supposed

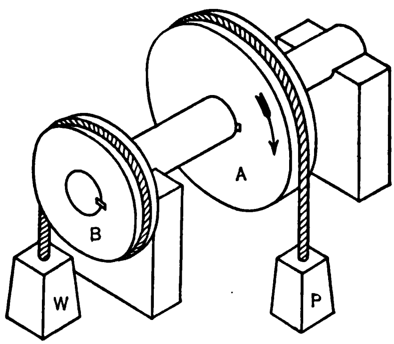


FIG. 5.

to be hung from a cord which is wound on the outside of wheel A and the weight W from another cord wound on the outside of wheel B. Suppose now that the shaft starts to turn in the direction shown by the arrow. Then the cord which supports P will be paid out, that is, will unwind, at a speed just equal to the periphery speed of A and the weight P will drop at that speed. At the same time the other cord will be winding on to the wheel B and the weight W will be rising at a speed equal to the periphery speed of B. If N represents the number of turns per second of the shaft, R

the radius of A, R_1 the radius of B, then the speed of $P = 2\pi RN$ and speed of $W = 2\pi R_1 N$, or

$$\frac{\text{Speed } P}{\text{Speed } W} = \frac{R}{R_1}, \quad \dots \dots \quad (5)$$

which is the same equation found when both points were on the same wheel.

Example. Let the diameter of $A = 12$ in. and the diameter of $B = 8$ in. and let the shaft turn $1\frac{1}{2}$ times per second. Then

$$\text{Speed } P = 2\pi \times 6 \times 1\tfrac{1}{2} = 56.55 \text{ in. per second.}$$
$$\text{Speed } W = 2\pi \times 4 \times 1\tfrac{1}{2} = 37.70 \text{ in. per second.}$$

Now, according to Eq. (5) these speeds should be in the ratio of 6 to 4 or 1.5 to 1, and if 56.55 is divided by 37.7, the result is equal to 1.5 except for the slight error due to carrying the figures to only two places of decimals.

18. Relation between Forces and Speeds. Suppose that in Fig. 5 it is assumed that there is no friction and that the weight of P and W are such that, if the shaft is at rest, the weights will just balance each other, or, if the shaft is caused to start turning in a given direction, the weights will allow it to keep turning at a uniform speed.

The *work* done by a force is equal to the force, expressed in units of force, multiplied by the distance through which the force acts, expressed in linear units. Now, if friction is neglected, the work obtained from a machine must equal the work put into it. Hence, in Fig. 5, if the falling weight P be considered as the force driving the machine, the work put into the machine is the weight P multiplied by the distance P falls. The work obtained from the machine is the weight of W multiplied by the distance W is raised.

Then the weight of P (in pounds or other weight units) multiplied by the distance P moves in a given length of time is equal to the weight of W (in the same kind of weight units)

multiplied by the distance it moves in the same time. If the shaft is assumed to make one turn, the distance moved by P and W are $2\pi RN$ and $2\pi R_1N$, respectively. Therefore,

$$P\times 2\pi RN = W\times 2\pi R_1N, \quad \ldots \quad (6)$$

or

$$\frac{\text{Weight of } P \text{ in lb.}}{\text{Weight of } W \text{ in lb.}} = \frac{R_1}{R}. \quad \ldots \quad (7)$$

Since from Eq. (5) the

$$\frac{\text{Linear speed of } P}{\text{Linear speed of } W} = \frac{R}{R_1},$$

and from Eq. (7)

$$\frac{\text{Weight of } P}{\text{Weight of } W} = \frac{R_1}{R},$$

Therefore,

$$\frac{\text{Weight of } P}{\text{Weight of } W} = \frac{\text{Speed of } W}{\text{Speed of } P}. \quad \ldots \quad (8)^*$$

19. Levers. The mechanical elements known as levers, in some form or other, are familiar to every one. They are subject to the same laws that have just been explained for revolving wheels. A lever is not ordinarily expected to make a complete turn or revolution like the wheel, but to swing or oscillate through some relatively small angle. This is not, however, an essential point.

The relation between a lever and a revolving wheel may be seen from the following discussion: The lever shown in Fig. 6, which turns about the axis C, may be assumed to be a piece cut out of the wheel which is shown by the dotted circle. Then if the lever turns through a certain angle in a unit time, the equations which have already been derived

* It must be always borne in mind that any equation such as Eq. (8) does not take friction into account.

for linear speed of points on a revolving wheel will apply equally well here to points A and B or any other points on the lever. In Fig. 6 the two arms on the lever are shown 180° with each other. This condition does not necessarily

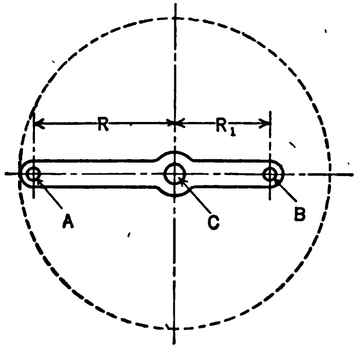


FIG. 6.

hold however, for the two arms may make any angle with each other from 180° as in Fig. 6 down to 0° as in Fig. 7. When the angle between the two arms is less than 90° as in

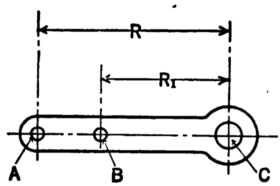


FIG. 7.

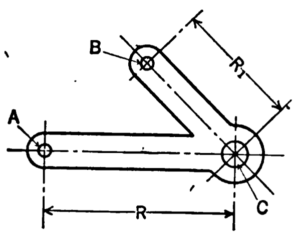


FIG. 8.

Figs. 7 and 8 it is often called a **bell crank lever**, and when the angle is more than 90° as in Figs. 6 and 9 it is often called a **rocker.** These terms are used rather loosely and somewhat interchangeably however.

In all these cases the following equation holds true:

$$\frac{\text{Linear speed } A}{\text{Linear speed } B}=\frac{\text{Distance of } A \text{ from axis}}{\text{Distance of } B \text{ from axis}}. \quad (9)$$

The two lever arms may be in the same plane as in Figs. 6 to 9 or they may be attached to the same shaft but lie in different planes as in Fig. 10.

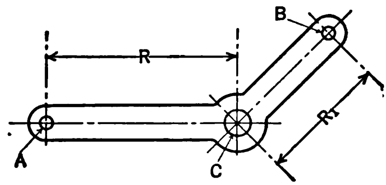


Fig. 9.

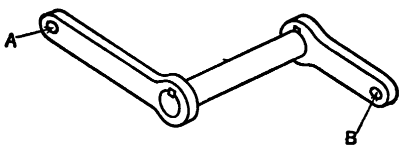


Fig. 10.

20. Right-handed and Left-handed Rotation. The foregoing laws are independent of the direction in which a wheel or lever is rotating. Much of the later work will involve direction, however, and it is well to get in mind the names ordinarily applied to the different directions. If a wheel or other body turns in *the direction of the hands of a clock* it is said to have *right-handed rotation,* or turn right-handed, and if it turns *anti-clockwise* it is said to turn *left-handed.*

CHAPTER III

TRANSMISSION OF MOTION BY MEANS OF CYLINDERS, CONES AND DISCS

21. Order of Treatment. In Chapter II were explained certain laws which apply to each of the mechanical units or elements of group 2, page 5. In the present chapter we shall consider the way in which these laws are taken account of when combining certain of the mechanical units into mechanisms (see § 2).

It is impossible to consider the various elements in the exact order in which they were named on page 5 because elements of different groups are often combined in the same mechanism.

In this chapter we shall deal with mechanisms made up of elements of group 1 and 2 in the following order:

1. A stationary piece and two or more cylinders
2. A stationary piece and two or more cones.
3. A stationary piece, cylinders and discs.

22. Cylinders Rolling Together without Slipping. External Contact. In Fig. 11 let A be a cylinder fast to the shaft S and B a cylinder fast to the shaft S_1. Assume that the shafts are held by the frame so that their centers are at a distance apart just equal to the sum of the radii of the two cylinders; that is, $R+R_1=C$. Then the surfaces will touch at P. Suppose also that the nature of the surfaces of the cylinders is such that, as they turn on their respective axes, there can be no slipping of one surface on the other. Then the surface speed of A must be equal to that of B, and A and B must turn in such directions relative to each other that the

line on A which is in contact with B is moving in the same direction as the line on B which it touches. (Notice the

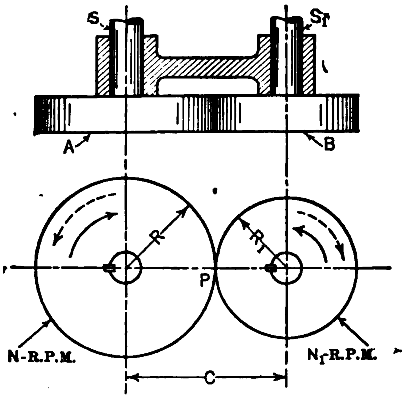


FIG. 11.

arrows in the figure, the full arrows belonging together and the dotted arrows together).

If A makes N r.p.m. and B makes N_1 r.p.m. then, from Eq. (2),

$$\text{Surface speed of } A = 2\pi RN,$$

and

$$\text{Surface speed of } B = 2\pi R_1 N_1.$$

Therefore, if the surface speed of A equals the surface speed of B,

$$2\pi RN = 2\pi R_1 N_1 \quad \text{or} \quad \frac{N}{N_1} = \frac{R_1}{R}. \quad . \quad . \quad . \quad (10)$$

Or, in other words, *the angular speeds of two cylinders which roll together without slipping are inversely proportional to the radii of the cylinders.*

This principle is fundamental and must be clearly understood.

23. Solution of Problems on Cylinders in External Contact. In Fig. 11 suppose C, N, and N_1 are known; required to find the diameters of the two cylinders. From Eq. (10),

$$\frac{R}{R_1}=\frac{N_1}{N} \quad \text{or} \quad R=\frac{R_1N_1}{N}.$$

It is known also that $R+R_1=C$. R and R_1 can, therefore, be found by solving these as simultaneous equations and, knowing the radii, the diameters can readily be found.

The same result may also be found by a simple graphical construction as follows: Draw the line SS_1, Fig. 12, making

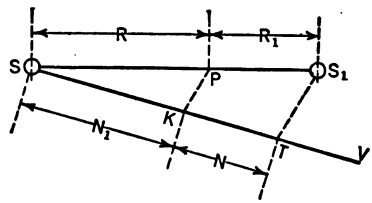


FIG. 12.

its length equal to the distance between the centers of the shafts (the same as C, Fig. 11). This would, in most cases, have to be drawn at some reduced scale. From S draw a line SV, making any angle with SS_1. From S lay off the distances SK equal to N_1 linear units and KT equal to N linear units. The line ST is then divided into two parts SK and KT such that $\frac{SK}{KT}=\frac{N_1}{N}$. Now connect T with S_1 and from K draw a line parallel to TS_1, cutting SS_1 at P. Then, from the similar triangles SKP and STS_1, $\frac{SP}{PS_1}=\frac{SK}{KT}=\frac{N_1}{N}$. Therefore, SP will be the radius R and PS_1 the radius R_1.

Example 1. Two shafts A and B are 16 in. on centers. A is to turn 50 times in a minute and B 150 times in a minute. What must be

the size of the cylinders to connect them if they are to turn in opposite directions?

Calculation. From Eq. (10).

$$\frac{\text{Radius of } A}{\text{Radius of } B}=\frac{\text{Turns of } B \text{ per minute}}{\text{Turns of } A \text{ per minute}}=\frac{150}{50}=\frac{3}{1},$$

or,

$$\text{Radius of } A=3\times\text{radius of } B.$$

Also

$$\text{Radius of } A+\text{radius of } B=16 \text{ in.}$$

Therefore,

$$3\times\text{radius of } B+\text{radius of } B=16 \text{ in.}$$

or,

$$4\times\text{radius of } B=16 \text{ in.}$$

Therefore,

$$\text{Radius of } B=4 \text{ in.}$$

and

$$\text{Radius of } A=3\times 4=12 \text{ in.}$$

Graphical Solution. In Fig. 13 draw the line AB equal to 16 in. at some reduced scale.

From A draw the line AV at any angle. Lay off AK equal to 150

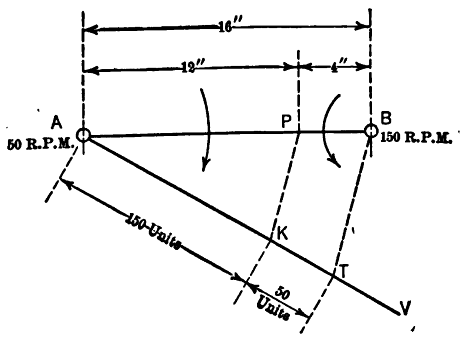


FIG. 13.

units. Lay off $KT=50$ of the same units. Join T with B and draw KP parallel to TB. Then BP will be found to measure 4 in. and AP 12 in., making proper allowance for the scale at which AB was drawn.

Example 2. A cylinder 18 in. diameter on a shaft A making 75 r.p.m. drives by rolling contact a cylinder on another shaft B, the second cylinder being $4\frac{1}{2}$ in. diameter. How fast does B turn if the shafts turn in opposite directions?

Calculation. $$\frac{\text{r.p.m. of } B}{\text{r.p.m. of } A}=\frac{\text{Diam. of cylinder on } A}{\text{Diam. of cylinder on } B}=\frac{18}{4\frac{1}{2}}=\frac{4}{1},$$

or

$$\text{r.p.m. of } B=4\times\text{r.p.m. of } A=4\times 75=300.$$

Graphical Solution. (Fig. 14.) Draw the line AB equal in length to the sum of the radii of the two cylinders $=\frac{18}{2}+\frac{4\frac{1}{2}}{2}=11\frac{1}{4}$ in.

On AB locate the point P 9 in. from A (therefore $2\frac{1}{4}$ in. from B). From B, the center of the cylinder whose speed is to be found, draw a

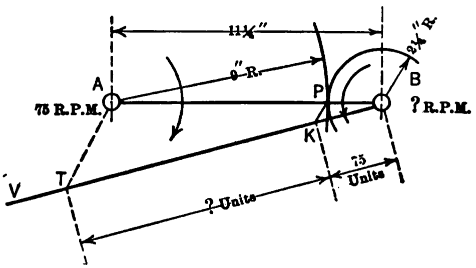


FIG. 14.

line BV at any angle and lay off on this line BK equal to 75 units (that is, speed of A). Join K with P and through A draw a line parallel to PK cutting BV at T. Then the units in KT will show the speed of B.

24. Cylinders Rolling Together without Slipping. Internal Contact. In Fig. 15, where the lettering corresponds to that of Fig. 11, the cylinder A is hollow with B inside it, so that the contact is between the inner surface of A and the outer surface of B. This is called *internal contact*. The same mathematical reasoning will apply here as in Fig. 11, and Eq. (10) will hold true. The distance between centers now, however, is equal to $R-R_1$ instead of $R+R_1$. The

two cylinders in Fig. 15 will turn in the same direction instead of in opposite directions.

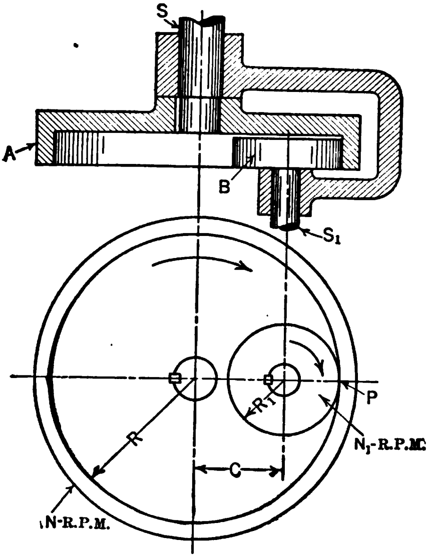


FIG. 15.

25. Solution of Problems on Cylinders in Internal Contact. In Fig. 15 if C, N and N_1 are known, to find the diameters of the cylinders.

From Eq. (10),

$$\frac{R}{R_1}=\frac{N_1}{N}.$$

It is also known that $R-R_1=C$. These may be solved as simultaneous equations to find R and R_1, and therefore the diameters.

The graphical solution of problems on cylinders rolling in internal contact is similar in principle to that shown in Fig. 12 for external contact. Fig. 16 shows the construction for internal contact.

Example 3. Two shafts A and B are 8 in. on centers and are to be connected by rolling cylinders to turn in the same direction; A to

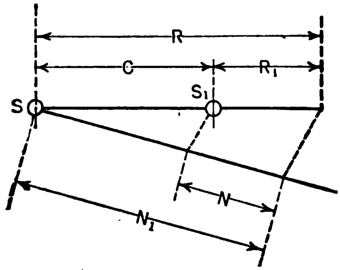


FIG. 16.

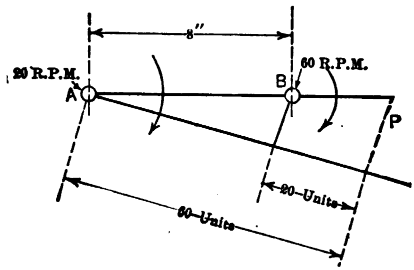


FIG. 17.

make 20 r.p.m. and B to make 60 r.p.m. Find the diameters of the cylinders.

Calculation. From Eq. (10),

$$\frac{\text{Rad. } A}{\text{Rad. } B}=\frac{\text{r.p.m. } B}{\text{r.p.m. } A}=\frac{60}{20}=\frac{3}{1},$$

or

$$\text{Rad. } A=3\times\text{rad. } B,$$

also

$$\text{Rad. } A-\text{rad. } B=8 \text{ in.}$$

$$3 \text{ rad. } B-\text{rad. } B=8 \text{ in.}$$

or

$$2 \text{ rad. } B=8 \text{ in.}$$

$$\text{Rad. } B=4 \text{ in.} \qquad \textit{Ans.}$$

and

$$\text{Rad. } A=3\times\text{rad. } B=3\times 4 \text{ in}=12 \text{ in.} \qquad \textit{Ans.}$$

Graphical Solution. Fig. 17 shows the graphical solution for this problem.

Example 4. A cylinder 24 in. diameter on a shaft A making 60 r.p.m. drives by rolling contact a cylinder on another shaft B, the second cylinder being 6 in. diameter. The shafts turn in the same direction. How fast does B turn and how far apart are the shafts?

Calculation. $$\frac{\text{Rev. } B}{\text{Rev. } A}=\frac{\text{Diam. cyl. on } A}{\text{Diam. cyl. on } B}=\frac{24}{6}=\frac{4}{1}.$$

$$\text{R.p.m. of } B=4\times\text{r.p.m. of } A=4\times 60=240 \text{ r.p.m.,}$$

and

$$\text{Dist. between centers}=\text{rad. } A-\text{rad. } B,$$

or

$$\text{Dist. between centers}=12-3=9 \text{ in.} \qquad \textit{Ans.}$$

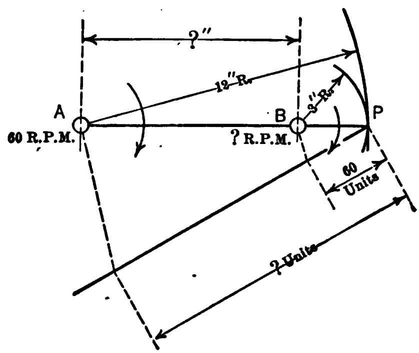


FIG. 18.

Graphical Solution. Fig. 18 shows the graphical solution of Problem 4.

26. Cones Rolling Together without Slipping. External Contact. In the preceding discussion relating to cylinders, the shafts were necessarily parallel. It is often necessary to connect two shafts which lie in the same plane but make some angle with each other. This is done by means of right cones as shown in Fig. 19 or frusta of cones as shown in

Fig. 20. The same reasoning applies to the ratio of speeds at the base of the cones as to the circles representing the cylinders in Fig. 11. That is,

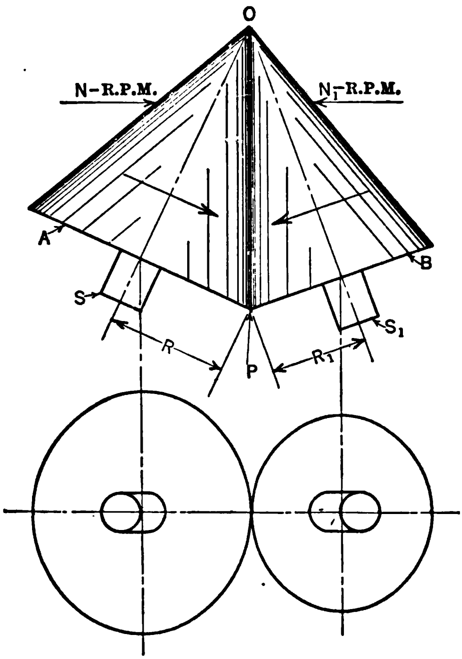


Fig. 19.

$$\frac{N}{N_1}=\frac{R_1}{R}. \qquad (11)$$

But

$$R_1=OP \sin POS_1$$

and

$$R=OP \sin POS.$$

Therefore,

$$\frac{R_1}{R}=\frac{OP \sin POS_1}{OP \sin POS}=\frac{\text{Sin } POS_1}{\sin POS}.$$

Substituting this expression in Eq. (11).

$$\frac{N}{N_1}=\frac{\text{Sin } POS_1}{\text{Sin } POS}. \qquad \qquad (12)$$

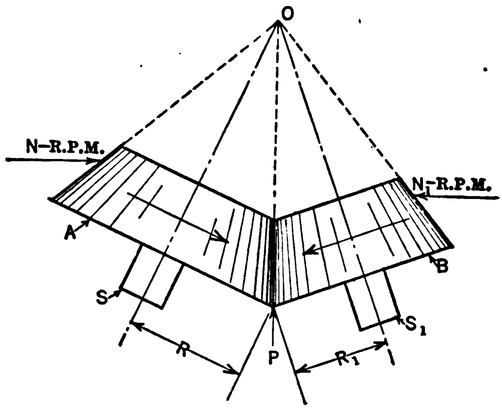


FIG. 20.

27. Solution of Problems on Cones in External Contact. Problems on rolling cones similar to those already discussed in connection with cylinders, may be solved by calculation, using Eq. (11) or Eq. (12). Such calculations are very likely to involve functions of the angle between the shafts and long trigonometric formulæ would then be necessary. The solution of problems on cones by means of a graphical construction is, therefore, preferable unless extreme accuracy is desired.

In Fig. 21, S and S_1 are two shafts which are to be connected by rolling cones to turn as indicated by the arrows. Their center lines meet at O. S is to make N r.p.m. and S_1 is to make N_1 r.p.m. Required to find the line of contact of two cones which will connect the shafts, and to draw a pair of cones.

Draw a line parallel to OA, on the side toward which its direction arrow points, at a distance from OA equal to N_1

units. Draw a similar line parallel to OB, N units distant from OB. These two lines intersect at K. A line drawn through O and K will be the line of contact of the required cones. Select any point P on OK and from P draw lines perpendicular to AO and BO meeting AO and BO at M and M_1, respectively. Produce these lines, making $MH=MP$

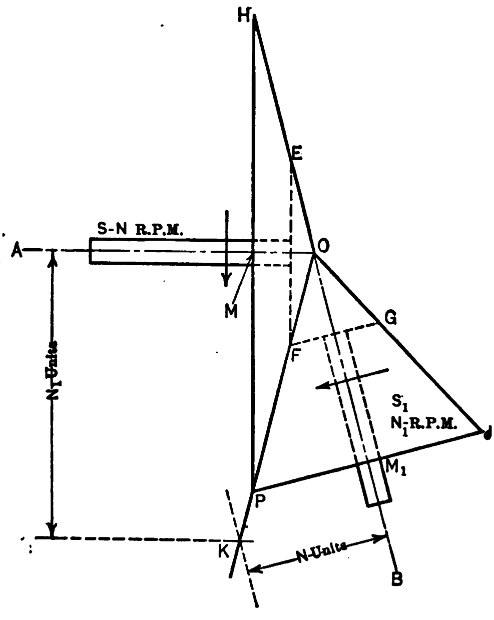


FIG. 21.

and $M_1J=M_1P$. Draw HO and JO. Then OPH and OPJ are cones of the proper relative sizes to connect S and S_1 to give the required speeds.

If the point P had been chosen nearer to O, the cones would have had smaller diameters at their bases but the ratio of the diameters would have been the same, or, if P had been chosen farther away from O, the bases would have been larger but still of the same ratio. If frusta of cones

are desired, the cones can be cut off anywhere, as shown by the dotted lines *FE* and *FG*.

Example 5. Two shafts S and S_1, Fig. 22, in the same plane, make an angle of 105° with each other. S is to turn 90 times per minute and

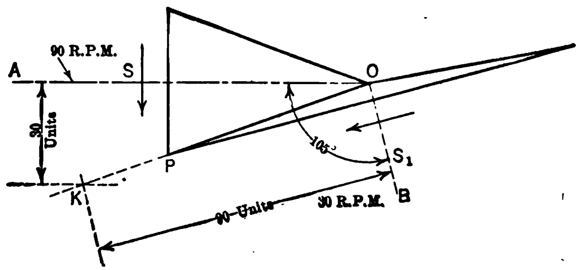


FIG. 22.

S_1 30 times per minute. A cone on A having a base $\frac{3}{4}$ in. diameter is to roll with a cone on B to give the required speeds, directions of rotation are to be as shown. To find the diameter of the base of the cone on B and to draw the cones.

Solution. Draw a line parallel to S 30 units distant from S, and a line parallel to S_1 90 units distant from S_1. These lines intersect at K; then KO is the element of contact. Since the base of the smaller cone is to be $\frac{3}{4}$ in. diameter, find a point P on OK which is $\frac{3}{8}$ in. from S. Through this point draw the bases of the cones perpendicular to S and S_1 in the same way as explained for Fig. 21.

28. Cones Rolling Together without Slipping. Internal Contact. Cones may be arranged for internal as well as external contact. This construction is not very often met in practice, but the necessity for it sometimes occurs and it is well to be familiar with the problem. Fig. 23 shows the principle involved. Corresponding points are lettered the same as in Fig. 19, and the same equations apply for finding the ratio of speeds of the two shafts. The same distinction as in the case of rolling cylinders holds true with regard to the opposite direction of rotation for external contact and the same direction for internal contact.

29. Solution of Problems on Cones Rolling in Internal Contact. No new principles are needed to solve problems

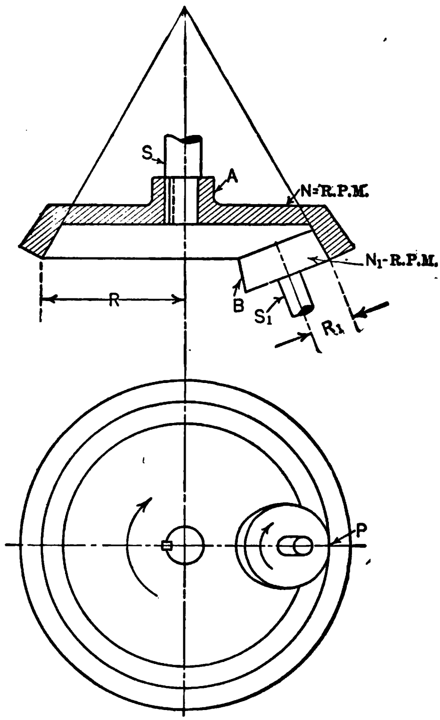


FIG. 23.

on cones in internal contact. The same methods used for cones in external contact are used for the internal contact.

30. Disc and Roller. A mechanism similar in action to the rolling cones is shown in Fig. 24. The shaft *S* has the disc *H* fast to it. On the shaft *T* is the roller *F* which has a key fitting into a spline in *T* but may be placed in any desired position by the "shipper" *G*. The roller is usually made of some fibrous material pressed tightly between two

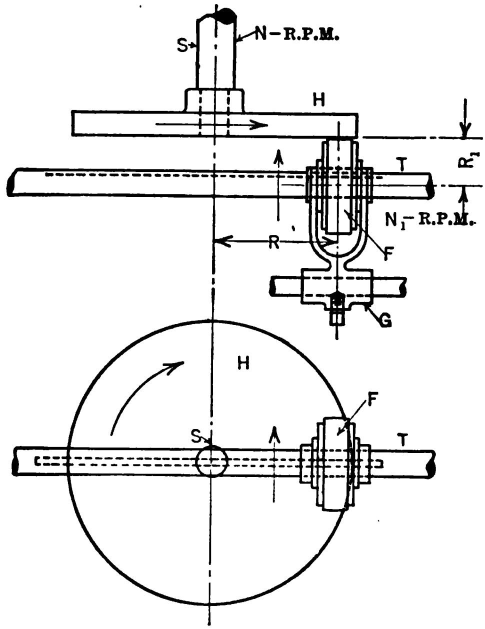


Fig. 24.

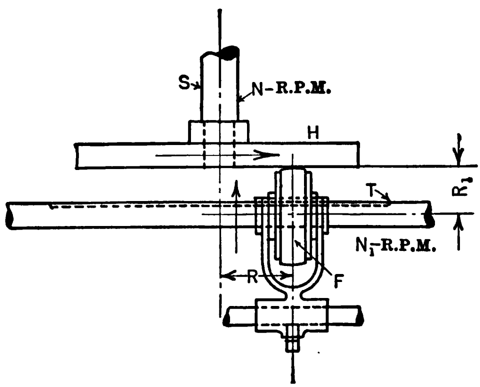


Fig. 25.

metal plates, and its outer surface may have a curved contour line as shown. If R is the distance from the center of S to the place where the roller touches the disc H, and R_1 is the radius of the roller, then, if there is sufficient friction between the surfaces to prevent slipping, $\frac{N}{N_1}=\frac{R_1}{R}$. If the roller is moved nearer the center of S as shown in Fig. 25,

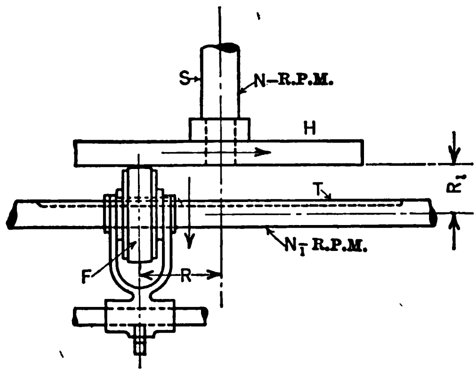


Fig. 26.

the distance R is decreased while R_1 remains constant. Therefore, if T is the driver running at a constant speed, the speed of S will be increased when the roller is placed nearer its center, or, if S is the driver, the speed of T will be decreased when the roller is placed nearer the center of the disc. If the roller is moved to the opposite side of the center of the disc as in Fig. 26, the direction of rotation of the driven shaft will be reversed.

CHAPTER IV

GEARS AND GEAR TEETH

31. Gear Drives. It will be seen from § 22 that if, in Fig. 27, the discs A and B are keyed to the shafts S and S_1 respectively, the angular speed of S is to the angular speed of S_1 as D_1 is to D, provided there is sufficient friction between the circumferences of the discs to prevent one slipping on the other. If the speed ratio must be exact, or if much power is

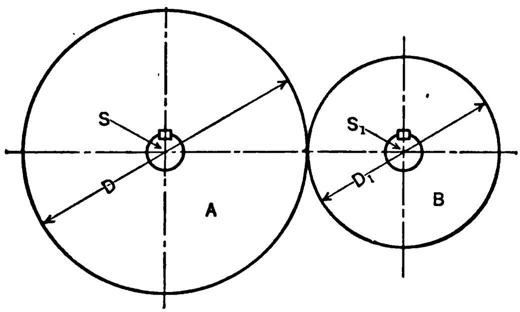


FIG. 27.

to be transmitted, a drive like this, depending solely upon friction, is not positive enough. To make sure that there shall be no slipping, wheels having teeth around their circumferences are substituted for the plain discs. Such a pair of wheels is shown in Fig. 28 and a similar pair in Fig. 29. In Fig. 28 the larger gear has 16 teeth and the smaller gear 12 teeth. Assume that the shaft S is being turned from some external source of power; the gear A, since it is keyed to S, will turn with it. Then the teeth on A will push the

teeth on B, a tooth on A coming in contact with a tooth on B and pushing that tooth along until the gears have turned so far around that those two teeth swing out of reach of each other. In order for B to make a complete revolution each

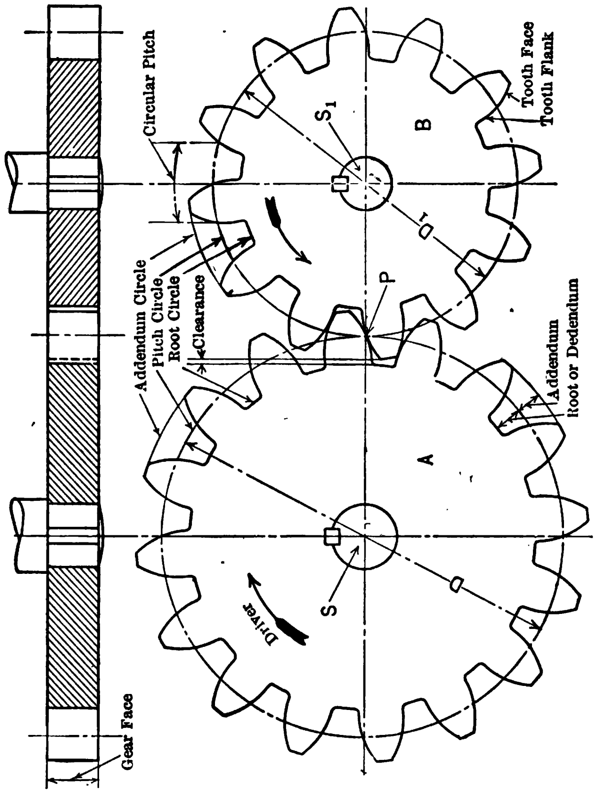


FIG. 28.

one of its 12 teeth must be pushed along thus past the center line. Therefore, while B turns once 12 of the teeth on A must pass the center line. Since A has 16 teeth in all, A will therefore make $\frac{12}{16}$ of a turn while B makes one turn. In other words, *the turns of A in a given time are to the turns of*

B in the same time as the number of teeth on B are to the number of teeth on A.

It is evident that the distance from the center of one tooth to the center of the next tooth on both gears must be alike in

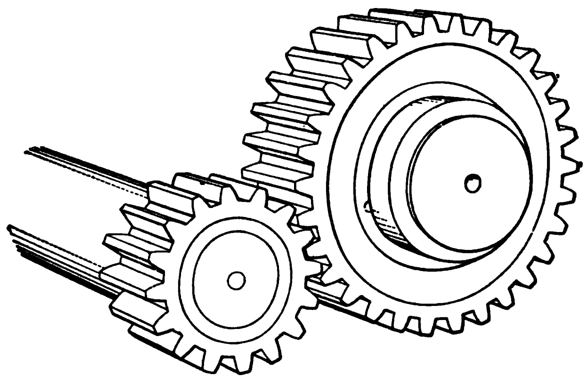


FIG. 29.

order that the teeth on one may mesh into the spaces on the other.

32. Pitch Circles and Pitch Point. Let a point P be found on the center line SS_1 such that $\frac{PS}{PS_1}=\frac{\text{Teeth on } A}{\text{Teeth on } B}$ and through this point draw circles about S and S_1 as centers. Call their diameters D and D_1. Then $D=2PS$ and $D_1=2PS_1$ Since, as shown above,

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } A}=\frac{\text{Teeth on } A}{\text{Teeth on } B},$$

therefore,

$$\frac{\text{Revolutions of } B}{\text{Revolutions of } A}=\frac{D}{D_1}.$$

That is, the two gears when turning will have the same speed ratio as would two rolling cylinders of diameters D and D_1. The point P which divides the line of centers of a pair of

gears into two parts proportional to the number of teeth in the gears is called the *pitch point.* The circle D, drawn through P with center at S is the *pitch circle* of the gear A and the circle D_1 is the *pitch circle* of the gear B.

33. Addendum and Root Circles. The circle passing through the outer ends of the teeth of a gear is called the *addendum circle* and the circle passing through the bottom of the spaces is called the *root circle.*

34. Addendum Distance and Root Distance. Length of Tooth. The radius of the addendum circle minus the radius of the pitch circle is the *addendum distance,* or, more commonly, the *addendum.* The radius of the pitch circle minus the radius of the root circle is the *root distance* or *root* or *dedendum.* The root plus the addendum is the *length of tooth.*

35. Face and Flank of Tooth. That portion of the tooth curve which is outside the pitch circle is called the *face of the tooth* or *tooth face.* This must not be confused with the term "face of gear" (§ 36). The part of the tooth curve inside the pitch circle is called the *flank of the tooth.*

36. Face of Gear. The length of the gear tooth in a direction parallel to the axis of the shaft is called the length of the face of the gear or *width of face* of the gear. (See top view, Fig. 28.)

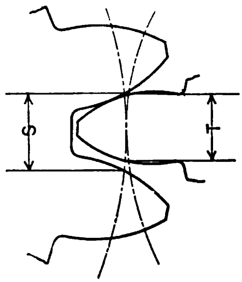


FIG. 30.

37. Clearance. The distance measured on the line of centers, between the addendum circle of one gear and the root circle of the other, when they are in mesh, is the *clearance.*

38. Backlash. When the width of a tooth, measured on the pitch circle, is less than the width of the space of the gear with which it is in mesh, the difference between the width of space and width of tooth is called the *backlash.* This is shown in Fig. 30 where S minus T is the backlash. Accurately made gears

rarely have any appreciable amount of backlash, but cast gears or roughly made gears require backlash.

39. Circular Pitch. The distance from the center of one tooth to the eenter of the next tooth, measured on the pitch circle, is called the *circular pitch*. This is, of course, equal to the distance from any point on a tooth which lies on the pitch circle to the corresponding point on the next tooth. See Fig. 28. *The circular pitch is equal to the width of tooth plus the width of the space between teeth, measured on the pitch circle.* The whole circumference of the pitch circle is equal to the circular pitch multiplied by the number of teeth, or the circular pitch is equal to the circumference of the pitch circle divided by the number of teeth. In Fig. 28 let T represent the number of teeth in the gear A and let C represent the circular pitch. Then,

$$C=\frac{\pi D}{T}. \qquad (13)$$

Two gears which mesh together must have the same circular pitch.

40. Diametral Pitch and Pitch Number. The term *diametral pitch* is used by different authorities to mean two different quantities. Most gear makers' catalogues and many books on gearing define diametral pitch as the number of teeth per inch of diameter of pitch circle. For example, if a gear has 24 teeth and the diameter of its pitch circle is 8 in., these authorities would say that the diametral pitch of the gear is 24 divided by 8 or is 3. This is also called the *pitch number.*

Other authorities define diametral pitch as the length of pitch diameter which the gear has per tooth, or the ratio of the diameter to the number of teeth. That is, referring again to the 24 tooth gear having a pitch diameter of 8 in., according to this latter definition the diametral pitch would be 8 in. divided by 24 or $\frac{1}{3}$ of an inch. Another name for this quantity is *module.*

Throughout this book the terms *diametral pitch* and *pitch number* will be used interchangeably, meaning the number of teeth per inch of pitch diameter, and the name *module* will be used for the amount of diameter per tooth.

If M represents the module and P.N. the pitch number or diametral pitch, T the number of teeth and D the pitch diameter, the above may be expressed in the form of equations as follows:

$$M=\frac{\text{Pitch diameter}}{\text{Teeth}}=\frac{D}{T}, \quad . \quad . \quad . \quad . \quad (14)$$

$$\text{P.N.}=\frac{\text{Teeth}}{\text{Pitch diameter}}=\frac{T}{D}. \quad . \quad . \quad . \quad (15)$$

41. Relation between Circular Pitch and Module.

From Eq. (14) $\qquad M=\frac{D}{T},$

and from Eq. (13)

$$C=\frac{\pi D}{T}.$$

Dividing Eq. (13) by Eq. (14),

$$\frac{C}{M}=\frac{\pi D}{T}\div\frac{D}{T}=\pi,$$

or

$$C=M\times\pi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

or

$$C=\frac{\pi}{\text{P.N.}}.$$

Or, in words, the circular pitch is equal to the module multiplied by 3.1416.

42. Angle and Arc of Action. The angle through which the driving gear turns while a given tooth on the driving gear is pushing the corresponding tooth on the driven gear is called the *angle of action* of the driver. Similarly, the angle through which the driven gear turns while a given one

of its teeth is being pushed along is called the angle of action of the driven gear.

In Fig. 31 a tooth M on the driving gear is shown (in full lines) just beginning to push a tooth N on the driven gear. The dotted lines show the position of the same pair of teeth when N is just swinging out of reach of M. While M has been pushing N, any radial line on the gear B, as for

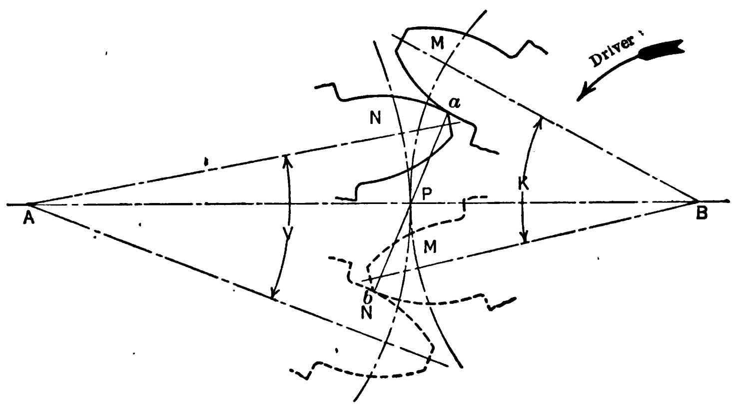


FIG. 31.

example, the line drawn through the center of the tooth M, has swung through the angle K, and any line on gear A has swung through the angle V. K is, therefore, the angle of action of the gear B and V is the angle of action of the gear A.

The arc of action is the arc of the pitch circle which subtends its angle of action.

The following equation holds true:

$$\frac{\text{Angle of action of driver}}{\text{Angle of action of driven gear}} = \frac{\text{Number of teeth on driven gear}}{\text{Number of teeth on driver}}. \quad . \quad (17)$$

The arc of action must never be less than the circular pitch.

43. The Path of Contact. Referring still to Fig. 31, the teeth as shown in full lines are touching each other at one point *a*. This point is really the projection on the plane of the paper of a line of contact equal in length to the width of the gear face (see § 35). In the position shown dotted the teeth touch each other at the point *b*. If the teeth were drawn in some intermediate position, they would touch at some other point. For every different position which the teeth occupy during the action of one pair of teeth they have a different point of contact. A line drawn through all the points at which the teeth touch each other (in this case the line *aPb*) is called the *path of contact.* This may be a straight line or a curved line, depending upon the nature of the curve which form the tooth-outlines. In all properly constructed gears the pitch point *P* is one point on the path of contact.

44. Law Governing the Shape of the Teeth. The curves which form the outline of the teeth on a pair of gears may, in theory at least, have any form whatever, provided they conform to one law, namely: *The line drawn from the pitch point to the point where the teeth are in contact must be perpendicular to a line drawn through the point of contact tangent to the curves of the teeth.*

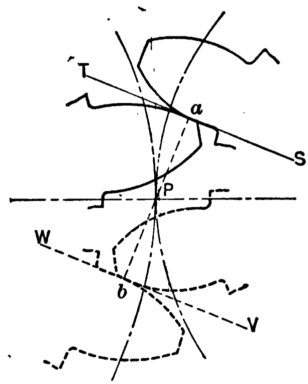


FIG. 32.

This is illustrated in Fig. 32. The teeth in the full line position touch each other at *a*. That is, the curves are tangent to each other at this point. The line *ST* is drawn tangent to the two curves at *a*. The curves must be made so that this tangent line is perpendicular to the line drawn from *a* to *P*. Similarly, in the dotted position the line *VW* which is tangent to the curves

at their point of contact b must be perpendicular to the line bP. This must hold true for all positions in which a pair of teeth are in contact in order that the speed ratio of the gears shall remain constant.

45. Obliquity of Action or Pressure Angle. The angle between the line drawn through the pitch point perpendicular to the line of centers, and the line drawn from the pitch point to the point where a pair of teeth are in contact is called the *angle of obliquity* of action or *pressure angle.* In some forms of gear teeth this angle remains constant while in other forms of teeth it varies.

The direction of the force which the driving tooth exerts on the driven tooth is along the line drawn from the pitch point to the point where a pair of teeth are in contact. The smaller the angle of obliquity the greater will be the component of the force in the direction to cause the driven gear to turn and the less will be the tendency to force the shafts apart. In other words, a large angle of obliquity tends to produce a large pressure on the bearings.

46. The Involute of a Circle. The form of the curve most commonly given to gear teeth is that known as the involute of a circle. Teeth properly constructed with this curve will conform to the law described in § 44. This curve and the method of drawing it will, therefore, be studied before considering the method of applying it to gear teeth.

In Fig. 33 the circle represents the end view of a cylinder around which is wrapped a cord, fastened to the cylinder at A and having a pencil in a loop at P. If now the pencil is swung out so as to unwind the cord from the cylinder, keeping it always taut, the curve which the pencil traces on a piece of paper on which the cylinder rests is known as an involute of the circle which represents the end view of the cylinder. All involutes drawn from the same circle are alike, but involutes drawn from circles of different diameters are different. The greater the diameter of the circle the flatter will be its involute.

In constructing the involute of a circle on the drawing board it is, of course, impossible actually to wrap a cord around the circle and draw the involute by unwinding the cord. Fig. 34 shows the method of constructing an involute on the drawing board. Suppose the involute is to be drawn

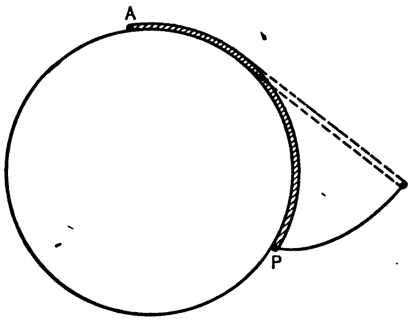


FIG. 33.

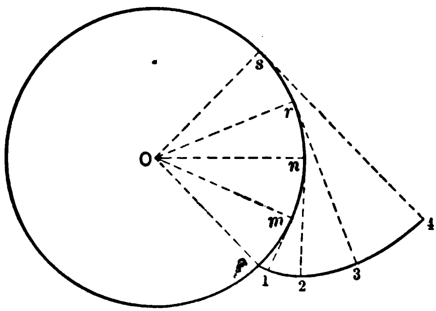


FIG. 34.

starting from any point p on the circle whose center is C. Set the dividers at any convenient short spacing; a distance which is about $\frac{1}{24}$ the circumference of the circle will give good results. Place one of the points of the dividers at p and space along on the circumference a few times, getting the equidistant points m, n, r, s. At each of these points draw radial lines and construct lines perpendicular to these

radii as shown. Each of these perpendiculars will then be tangent to the circle at one of the points. Taking care that the setting of the dividers remains unchanged, lay off one space $m1$ on the tangent at m. On the next line, which is tangent at n lay off from n the same distance twice, getting the point 2. From r lay off the distance three times, getting the point 3; and so on until points are found as far out as desired. A smooth curve drawn through these points with a French curve will be a very close approximation to the true involute,—close enough for all practical purposes if the work is done carefully.

47. Application of the Involute to Gears. In Fig. 35 let A and B be the centers of two gears whose pitch circles are

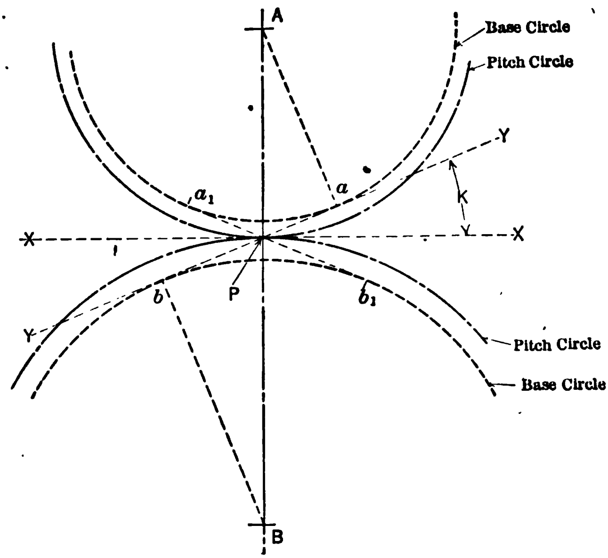


FIG. 35.

tangent at P. Through P draw a line XX perpendicular to the line of centers AB and another line YY making an angle K with XX. From A draw a line Aa perpendicular to YY

and from B draw Bb also perpendicular to YY. Then Aa and Bb will be the radii of circles drawn from A and B respectively, tangent to YY. These circles may be called *base circles.* Now, from geometry the triangle AaP is similar to the triangle BbP, therefore, $\frac{Aa}{AP}=\frac{Bb}{BP}$. That is, the radii of the base circles are in the same ratio as the radii of the pitch circles. Therefore, since

$$\frac{\text{Angular speed of } A}{\text{Angular speed of } B}=\frac{BP}{AP},$$

it follows that

$$\frac{\text{Angular speed of } A}{\text{Angular speed of } B}=\frac{Bb}{Aa}.$$

If now the tooth outlines on the gear A are made involutes of the circle whose radius is Aa and those on B involutes of the circle whose radius is Bb, it can be shown that a tooth on A will drive a tooth on B in such a way that at all times the angular speed of A will be to the angular speed of B as Bb is to Aa. The same ratio of speeds would hold if B were the driver. The teeth would always be in contact at a point on the line aPb or at a point on a_1Pb_1. The path of contact in gears having involute teeth is, therefore, a straight line and the angle of obliquity or pressure angle is constant. That is, the direction of the force which the driving tooth exerts on the driven tooth is the same at all times.

48. To Draw a Pair of Involute Gears. Suppose that it is required to draw a pair of involute gears 4-pitch, 16 teeth in the driver and 12 teeth in the driven gear; addendum on each to be $\frac{1}{4}$ in. and dedendum $\frac{9}{32}$ in.; pressure angle $22\frac{1}{2}°$.

One-half of one gear and 4 teeth on the other will be drawn. In Fig. 36 draw a center line and on this line choose a point S which is to be the center of the driving gear. To find the distance between centers and thus locate the center of the other gear, first find the pitch diameter of each.

Since the driver has 16 teeth and is 4-pitch (that is it has 4 teeth for every inch of pitch diameter), its pitch diameter must be 16÷4 or 4 in. In like manner the diameter of the other gear is 12÷4 or 3 in. The distance between centers

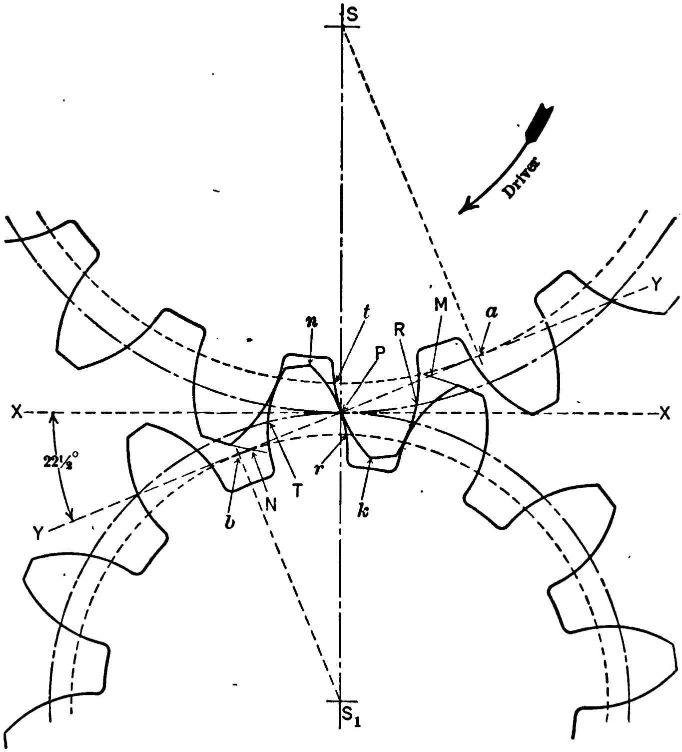


FIG. 36.

must be equal to the radius of the driver plus the radius of the driven gear and is, therefore, $2+1\frac{1}{2}$ in. or $3\frac{1}{2}$ in. Measure off the distance SS_1 equal to $3\frac{1}{2}$ in. and S_1 is the center of the driven gear. Next, locate the pitch point P, 2 in. from S or $1\frac{1}{2}$ in. from S_1, and through P draw arcs of circles with S

and S_1 as centers. These arcs are parts of the pitch circles of the two gears. Through P draw the line XX perpendicular to the line of centers and draw the line YY making an angle of $22\frac{1}{2}°$ with XX. From S and S_1 draw lines perpendicular to YY meeting it at a and b. With radii Sa and S_1b draw the base circles. Draw the addendum circle of the upper gear with S as a center and radius equal to the radius of the pitch circle plus the addendum distance. This will be $2\frac{1}{4}$ in. In similar manner draw the addendum circle of the lower gear with a radius $1\frac{3}{4}$ in. Draw the root circle of the upper gear with S as a center and radius $2-\frac{9}{32}$ in. (that is pitch radius−dedendum.) Find the root circle of the lower gear in a similar way. We are now ready to construct the teeth themselves.

From the point a space off on the base circle the arc at equal in length to the line aP and from t, thus found, draw the involute of the base circle of the upper gear as described for Fig. 35. tPk is the curve thus found. In a similar manner find the point r such that arc br is equal in length to the line bP and from r draw the involute rPn of the lower base circle.

The shape of the tooth curves having been found in this way, the next step is to find the width of the teeth on the pitch circles and draw in the rest of the curves. Since the gears are 4 pitch, the circular pitch is $\frac{1}{4}\times 3.1416=.7854$ in. and if the width of the tooth is one-half the circular pitch, as is usually the case, the width of the tooth on each gear must be $\frac{1}{2}\times .7854$ or .39 in. nearly. Therefore, lay off the arc PR equal to .39 in. and through R draw an involute which is a duplicate of the curve tPk except that it is turned in the reverse direction. Similarly make PT equal .39 and draw an involute through T which is a duplicate of the curve nPr. These curves can be transferred to the new positions by means of templets, it being unnecessary to construct the curve more than once. The part of the tooth outlines below the base circles may be made radial lines with small fillets

at the bottom corners. One tooth on each gear has now been completed and other teeth may be drawn like these by means of templets.

If the larger gear is the driver and turns in the direction indicated by the arrow, the path of contact is the line MPN.

49. Limits of Addendum on Involute Gears. Fig. 37 shows one tooth on each of a pair of 4-pitch gears of 18 and 24 teeth respectively. The addendum arcs of the teeth, shown in full lines, are such that the addendum distance is equal to $\frac{7}{8}$ the module ($\frac{7}{32}$ in.). If for any reason it is desired to redesign these gears with longer teeth, that is, with larger addendum circles, it will be necessary to know how long the teeth can be made without causing trouble. The tooth on B can be increased in length until the addendum circle passes through the point a, where the line of obliquity YY is tangent to the base circle of A. If the tooth is made longer than this limit, interference will result unless some special form of curve is constructed in place of the involute for the outer end of the tooth. The tooth on A might be lengthened until the addendum circle passed through the point of tangency b except for the fact that there is another limit to the addendum which sometimes has to be considered. The maximum addendum here is limited by the intersection of the two sides of the tooth giving a pointed tooth. It is evident that no further increase in addendum is here possible.

50. Standard Proportions. There is no one standard governing the relations between pitch, addendum, clearance, etc. Two methods of proportioning the teeth may be mentioned which, for convenience, will be referred to as the **Brown & Sharpe Standard** and the **American Society of Mechanical Engineers Standard.** The Brown & Sharpe standard represents the proportions ordinarily used by the Brown & Sharpe Manufacturing Company. Their practice is to modify the form of the tooth curves slightly at the

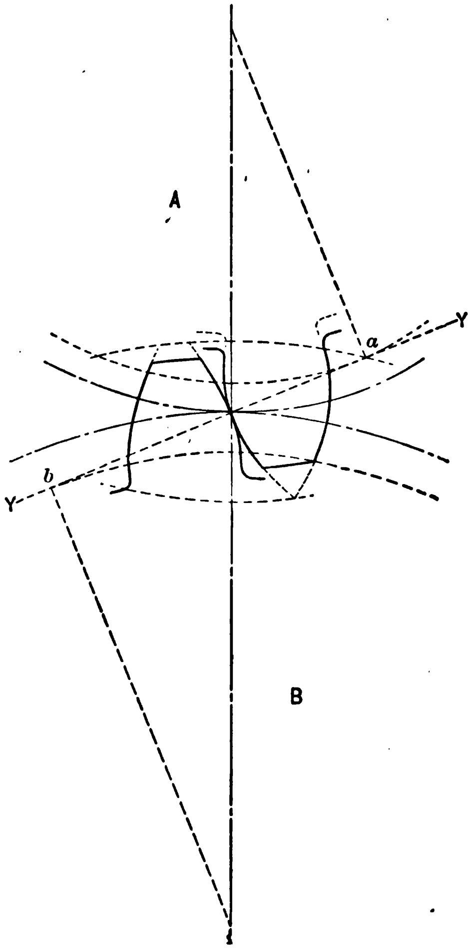


FIG. 37.

end as experience has shown them to be desirable. The A.S.M.E. standard is that proposed in a majority report of a committee appointed to recommend a standard which would be desirable for general adoption.

The following tables give the proportion for the two standards.

TABLE I.—BROWN & SHARPE STANDARD FOR INVOLUTE GEARS

Angle of obliquity (pressure angle)	$14\frac{1}{2}°$
Addendum	Equal to module
Clearance	Approximately $\frac{1}{6}$ module
Dedendum or root	Approximately $1\frac{1}{6}$ module

TABLE II.—A.S.M.E. STANDARD (PROPOSED) FOR INVOLUTE GEARS

Angle of obliquity (pressure angle)	$22\frac{1}{2}°$
Addendum	$\frac{7}{8}$ module
Clearance	$\frac{1}{8}$ module
Dedendum or root	Equal to module

51. Rack. A rack is a gear whose pitch line is a straight line, as shown in Fig. 38. The gear which meshes with a

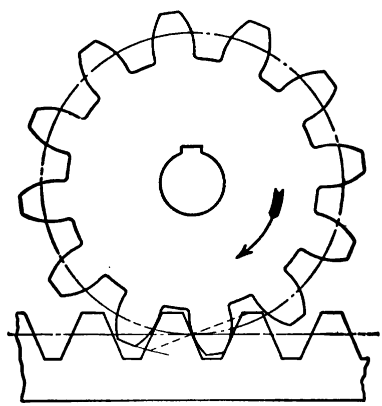


FIG. 38.

rack is often called a pinion. If the pinion turns on a fixed axis, the rack will be moved along in a straight line, suitable

guides being necessary. The rack may be the driver and by moving along cause the pinion to turn on its axis. If the rack is fixed in position and the pinion is caused to turn, it will roll along on the rack like a wheel rolling on a track. The tooth outlines of an involute rack are straight lines perpendicular to the line of pressure.

52. Annular Gear. Gears may run in internal contact, corresponding to the internal contact of cylinders (§ 24).

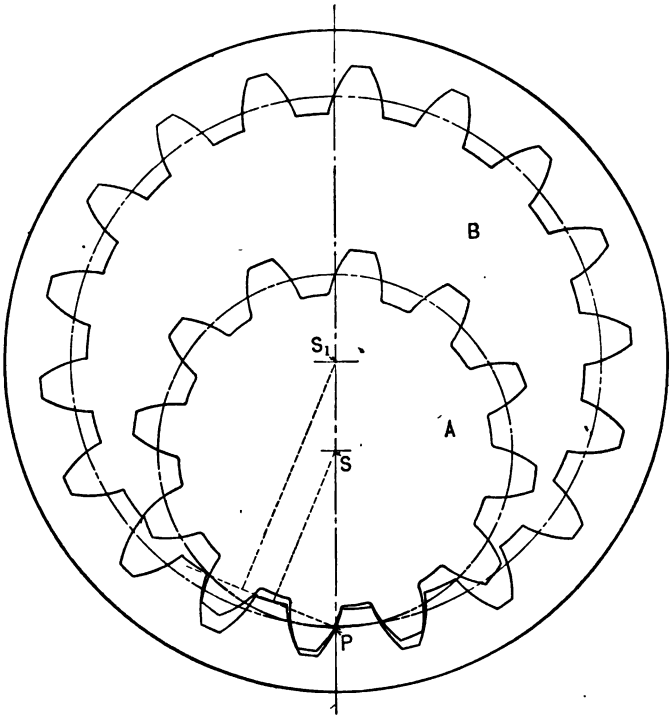


Fig. 39.

Fig. 39 shows such a pair of gears. The gear *B* consists of a hollow cylinder with teeth on its inner surface and is called an *annular gear*. The principles already brought out with

reference to gears working in external contact apply equally well in the case of a pinion and annular. P is the pitch point and the following equation holds true.

$$\frac{S_1P}{SP} = \frac{\text{Teeth on } B}{\text{Teeth on } A}.$$

53. Bevel Gears. A pair of bevel gears bears the same relation to a pair of rolling cones (§ 26) that a pair of spur gears bears to rolling cylinders.

Fig. 40 shows two bevel gears meshing together. Here

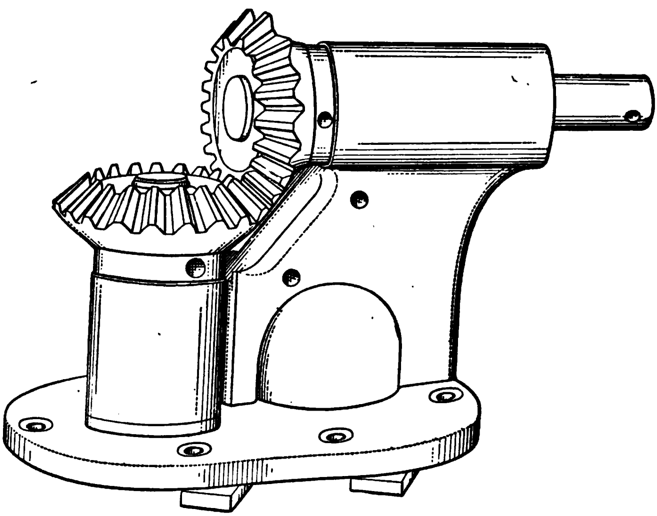


FIG. 40.

as in the case of spur gears, the angular speeds are inversely proportional to the number of teeth.

The pitch circle of a bevel gear is the base of the cone which the gear replaces.

54. To Draw the Blanks for a Pair of Bevel Gears. A convenient way to gain an understanding of the principle of bevel gear design will be to study the method of drawing

the blanks from which a pair of bevel gears is to be cut. Let it be assumed that a 6-pitch, 12-tooth gear is to mesh with an 18-tooth gear, the axes to intersect at 90°. Start

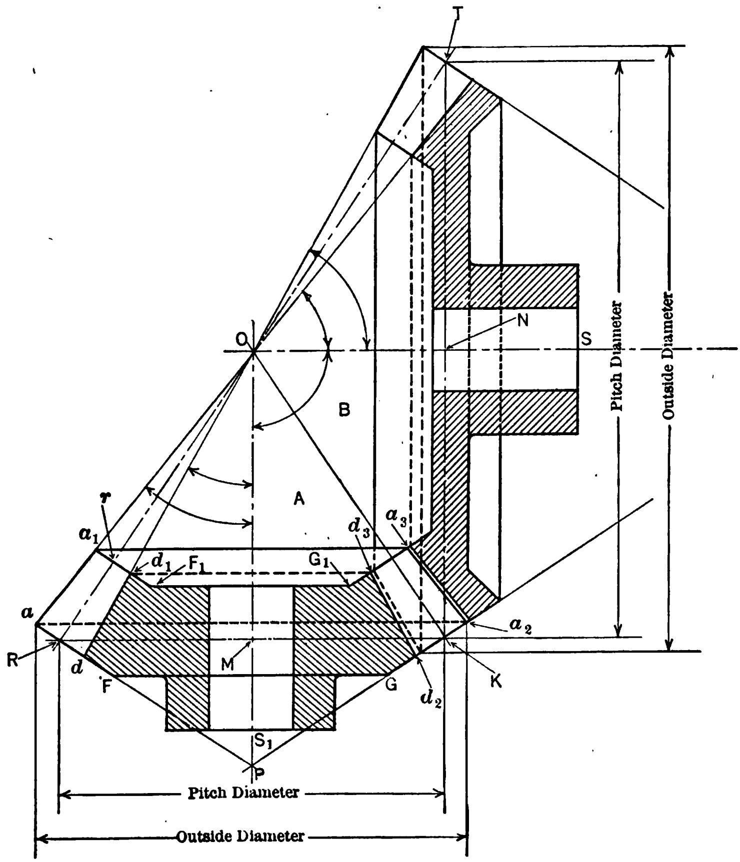


FIG. 41.

with the point O, Fig. 41, as the point of intersection of the two axes. Draw OS and OS_1 making the required angle (in this case 90°). These are the center lines of the shafts.

Assume that the 12-tooth gear is to be on S_1. Call this gear A and the 18-tooth gear B. Since A has 12 teeth and is 6-pitch, its pitch diameter, that is, the diameter of the base of its pitch cone (§ 53), is $12 \div 6$ or 2 in. In like manner the pitch diameter of B is $18 \div 6$ or 3 in. From O measure along OS_1 a distance OM equal to the pitch radius of B ($1\frac{1}{2}$ in.) and, through M, thus found, draw a line perpendicular to OS_1. In like manner make ON equal to the pitch radius of A and draw a line through N perpendicular to OS. These lines intersect at K. Make MR equal to MK and NT equal to NK. From R, K, and T draw lines to O. Then the triangle ORK is the projection of the "pitch cone" of the gear A and OTK that of the "pitch cone" of B. It will be noticed that the above construction is the equivalent of that for rolling cones given in § 27. Next, draw through K a line perpendicular to OK meeting OS_1 at P and OS at a point H not shown in the figure. Draw a line from H through T and from P through R. The cone represented by the triangle THK is called the *normal cone* of the gear B and that represented by the triangle RPK is called the normal cone of A.

From R lay off Ra equal to the addendum that is to be used on gear A (this is determined by the same considerations that would be used for the dedendum on a spur gear). Along RO lay off Rr equal to the desired length of gear face (§ 35). Through r draw a line parallel to PR. From a draw a line to O meeting this parallel at a_1. Through a draw parallel to RK meeting PHK at a_2. From a_1 draw parallel to RK meeting a line drawn from a_2 to O at a_3. Lay off along RP the distance Rd equal to the ~~addendum~~ and draw from d toward O meeting a_1r at d_1. Find d_2 and d_3 in the same way that a_2 and a_3 were found. The figure add_1a_1 represents the tooth. The dimensions of the hub and the position of lines FG and F_1G_1 may be made anything that is desirable.

55. Drawing the Teeth on Bevel Gears. Fig. 42 shows

a method of drawing the teeth on a pair of bevel gears. A complete description of the process will be omitted, the main steps being evident from the figure. Attention is

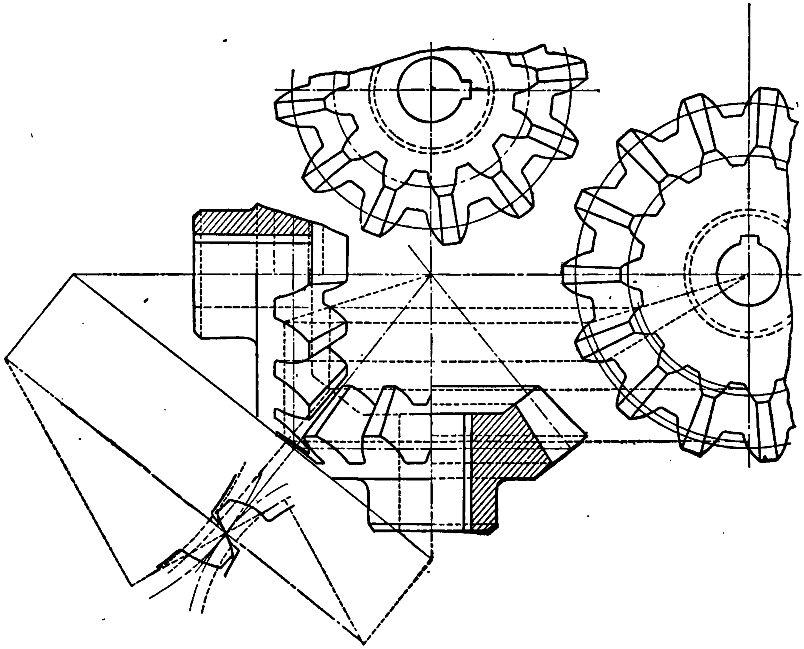


FIG. 42.

called to the fact that the circles used as pitch circles in the design of the teeth are portions of the development of the bases of the pitch cones.

56. Spiral Gears and Helical Gears. A pair of spiral gears is shown in Fig. 43. Here the shafts are neither parallel nor intersecting. The teeth wind round the outside of the gears in helical curves. When gears with helical teeth are used to connect parallel shafts, they are ordinarily called *helical gears,* and when used on non-parallel shafts they are called *spiral gears.* A full discussion of the theory of spiral

gears is beyond the scope of the present book and a few of the fundamental principles only will be touched upon. If the reader wishes to go more fully into the subject, he can

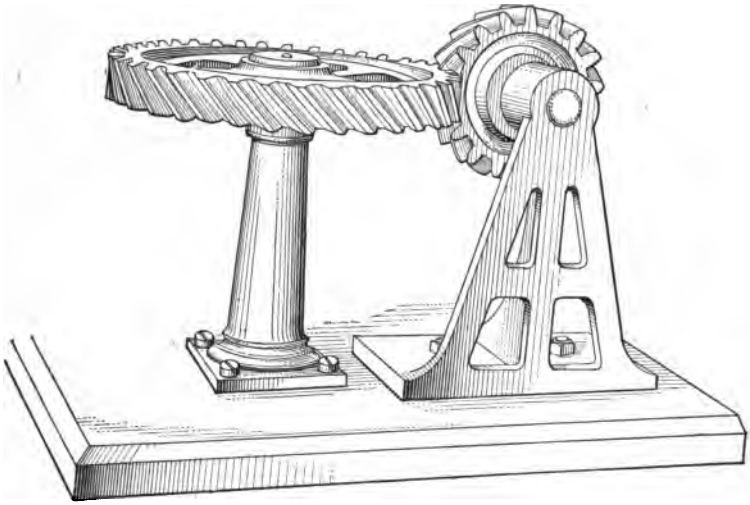


FIG. 43.

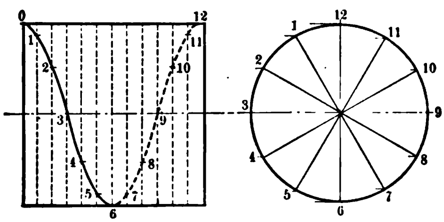


FIG. 44.

readily find, in any good library, books devoted entirely to the subject of gearing in which spiral gears are fully discussed.

57. The Helix; Its Construction and Properties. A helix is a curve wound around the outside of a cylinder or

cone advancing uniformly along the axis as it winds around. The nature of the curve and the method of drawing it may be understood from a study of Fig. 44.

The *angle of a helix* is the angle which a straight line tangent to the helix at any point makes with an element of the cylinder. This angle is the same for all points on the helix. Two helices are said to be normal to each other when their tangents drawn at the point where the helices intersect, are perpendicular to each other.

58. Speed Ratio of Spiral Gears. In Fig. 45 let S be the center line of the shaft on which is the driver in a pair of

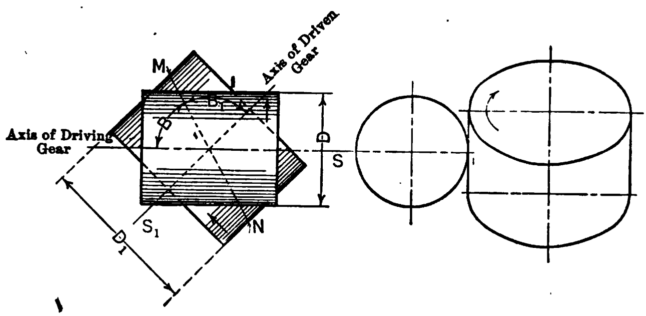


FIG. 45.

spiral gears, and S_1 the center line of the shaft on which is the driven gear. The line MN represents the line tangent to the helical pitch line of a pair of teeth at their point of contact. Then B is the angle of the helix on the driving gear and B_1 the angle of the helix on the driven gear. D is the pitch diameter of the driver and D_1 the pitch diameter of the driven gear.

The following formula can be deduced by trigonometry:

$$\frac{\text{Angular speed of driven gear}}{\text{Angular speed of driving gear}} = \frac{D \cos B}{D_1 \cos B_1}. \quad (18)$$

It can also be shown that

$$\frac{\text{Angular speed of driven gear}}{\text{Angular speed of driving gear}}$$

$$= \frac{\text{Number of teeth on driving gear}}{\text{Number of teeth on driven gear}}. \quad . \quad (19)$$

In connection with Eq. (19) it should be noted that the numbers of teeth on a pair of spiral gears are not proportional to the pitch diameters. This is true because the teeth wind around the circumference instead of running parallel to the axis.

59. Cycloidal Gears. At the present time almost all gears are based on the involute form of tooth already described. Formerly the curves used for the outlines of the teeth were cycloidal curves. This system has largely gone out of use except in special cases. A full discussion of this system can be found in almost any treatise on gearing. It does not seem to the authors to be of sufficient importance to be included here

CHAPTER V

BELTS, ROPES AND CHAINS

60. Belt Drives. It has been seen, in a previous chapter, that two shafts may be connected by cylinders in contact with each other, or by gears, which are the equivalent of rolling cylinders. If the shafts which are to be connected are so far apart that it is not possible or convenient to use cylinders large enough to be in direct contact with each other, some other means of connection must be provided. The usual method in such a case is to put cylinders of the proper relative diameters on the shafts, and stretch over the cylin-

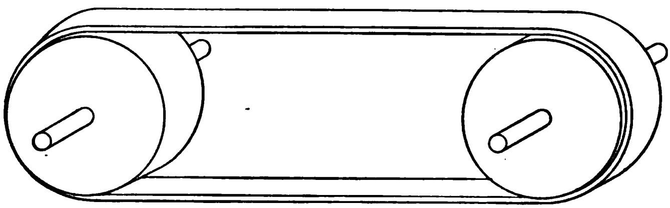


FIG. 46.

ders a flexible band to form the connection. A simple example of this is shown in Figs. 46 and 47; Fig. 47 being the orthographic projections of the same shafts shown pictorially in Fig. 46.

The connection may be a flat band or **belt** of leather, cotton, or other fiber, such as shown in the above figures, or it may be a **rope** either of wire or of any of the ordinary materials of which rope is made, or it may be a **chain.** The cylinders must have properly shaped surfaces to work to the

best advantage with the particular kind of connector used. In the case of the flat belt the cylinders are called **pulleys,**

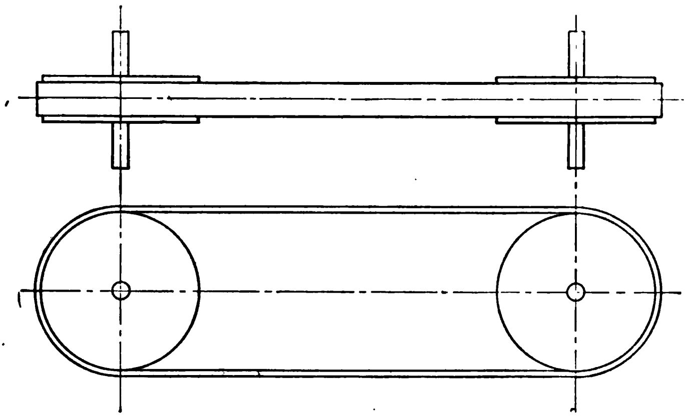


Fig. 47.

in the case of rope connectors the cylinders are often called **sheaves,** or sheave pulleys, and when used with a chain they are called **sprocket wheels.**

61. Pulleys for Flat Belt. Fig. 48 is a picture of a medium-sized plain pulley such as is used for leather and other flat belts. The details of construction, of course, vary with the size of pulley and the conditions under which it is used. One feature is almost always found in pulleys when the belt is expected to remain always at the center of the pulley. This is the feature known as *crowning.* The surface of the pulley, instead of being cylindrical, is slightly larger in diameter at the center than at the two edges, as shown in Figs. 49 and 50. The amount of crowning is exaggerated in these figures. In Fig. 49 the pulley has the form of two frusta of cones placed base to base, and in Fig. 50 the contour line is curved. In either case the object is the same, namely to keep the belt at the center of the pulley. A belt tends to run to the place where it is tightest; conse-

quently, where the diameter of the pulley is larger at the middle the belt tends to run at the middle. The reason for

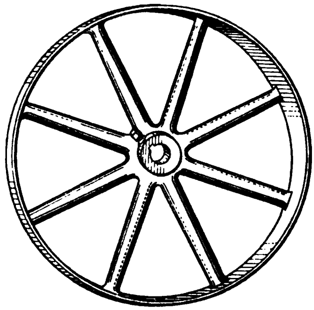


FIG. 48.

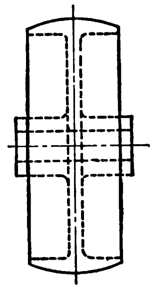


FIG. 49.

this action of the belt may be explained as follows: Let Fig. 51 represent a pulley very much wider than the belt which is to run on it, with the crowning very much exaggerated. Suppose the belt is stretched over this pulley with considerable tension. The pull on the belt tends to make it hug the surface of the pulley somewhat as indicated by the full lines,

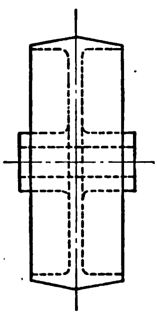


FIG. 50.

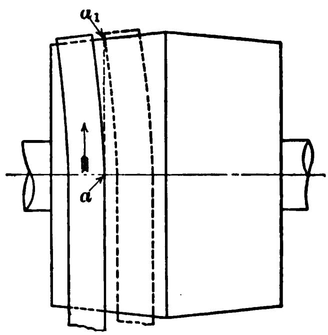


FIG. 51.

although the stiffness of the belt does not allow it to quite occupy this position. Any point, as a, on the pulley, which

is in contact with the belt, is moving in the direction of the arrow, and, after a part of a turn, will arrive at a_1. Since the belt adheres to the surface of the pulley, with comparatively little slipping, the belt will tend to go to the position shown dotted. This process continues until the belt has climbed to the highest part of the pulley.

62. Speed Ratio of Shafts Connected by a Belt. In Fig. 52 let the diameter of the pulley on shaft S be D inches and

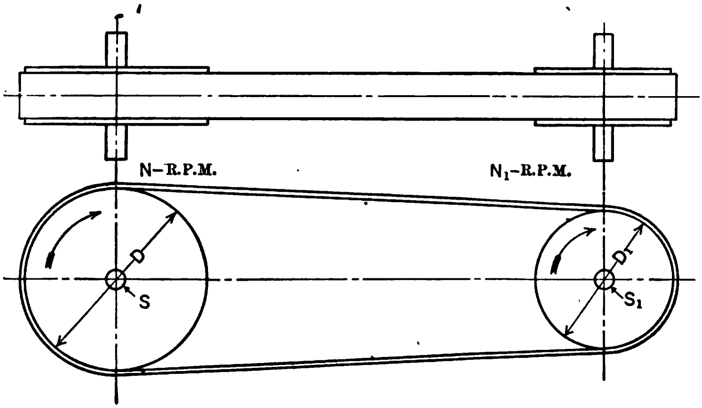


FIG. 52.

the diameter of the pulley on S_1 be D_1 inches. Also let N represent the r.p.m. of S and N_1 represent the r.p.m. of S_1.

Then from the law expressed in equation (2)

$$\text{Surface speed of pulley on } S = \pi DN,$$

and

$$\text{Surface speed of pulley on } S_1 = \pi D_1 N_1.$$

If the belt adheres to the surface of both pulleys so that there is practically no slipping, the surface speeds of both pulleys will be the same.

That is

$$\pi DN = \pi D_1 N_1.$$

Therefore,

$$\frac{N}{N_1}=\frac{D_1}{D}, \qquad \dots\dots\dots \qquad (20)$$

or, in other words, *the speeds of two shafts connected by a belt running over pulleys are inversely as the diameters of the pulleys.* In theory the diameters used in the preceding discussion should be the diameters of circles half the thickness of the belt outside the real pulleys and known as the *effective diameters.* This degree of refinement is unnecessary, however, and, in all practical work the diameters considered are the actual pulley diameters.

63. Solution of Problems on Speed of Shafts Connected by a Belt.

Example 6. Assume that a shaft A makes 360 r.p.m. On A is a pulley 24 in. diameter belted to a pulley 36 in. diameter on another shaft B.

From Eq. (20)

$$\frac{\text{Speed of } A}{\text{Speed of } B}=\frac{\text{Diam. of pulley on } B}{\text{Diam. of pulley on } A}.$$

Substituting the known values, this equation becomes

$$\frac{360}{\text{Speed of } B}=\frac{36}{24}.$$

Therefore,

$$\text{Speed of } B=\frac{24}{36}\times 360=240 \text{ r.p.m.}$$

Example 7. Suppose a shaft A making 210 r.p.m. is driven by a belt from a 30-in. pulley on another shaft B which makes 140 r.p.m.; to find the size of the pulley on A. Using the principle of Eq. (20),

$$\frac{\text{Speed of } A}{\text{Speed of } B}=\frac{\text{Diam. of pulley on } B}{\text{Diam. of pulley on } A}.$$

Therefore,

$$\frac{210}{140}=\frac{30}{x} \quad \text{or} \quad x=\frac{30\times 140}{210}=20 \text{ in.}$$

Then a 20-in. pulley is required on A.

64. Relative Direction of Rotation. The relative directions in which the pulleys turn depend upon the manner in which the belt is put on the pulleys. The belt shown in

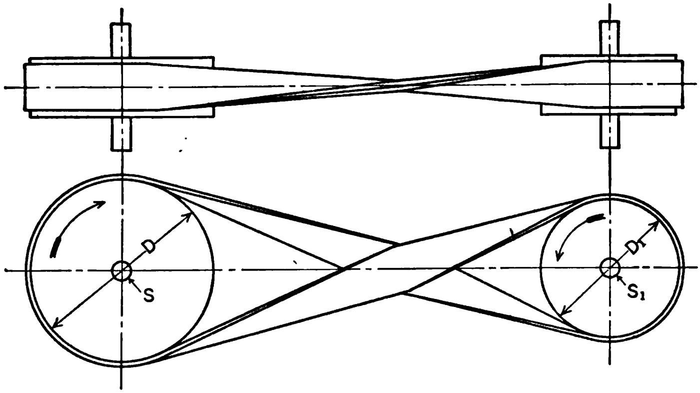


FIG. 53.

Fig. 52 is known as an **open belt** and the pulleys turn in the same direction as suggested by the arrows. The belt shown in Fig. 53 is known as a **crossed belt** and the pulleys turn in opposite directions as indicated.

65. Kinds of Belts. The material most commonly used for flat belts is leather. For some kinds of work, however, belts woven from cotton or other similar material are used. When the belt is to be run in a place where there is much moisture, it may be made largely of rubber properly combined with fibrous material in order to give strength.

Leather belts are made by gluing or riveting together strips of leather cut lengthwise of the hide, near the animal's back. If single thicknesses of the leather are fastened end to end, the belt is known as a **single belt** and it is usually about $\frac{3}{16}$ in. thick. If two thicknesses of leather are glued together, flesh side to flesh side, the belt is known as a **double belt** and is from $\frac{5}{16}$ to $\frac{3}{8}$ in.

thick. The manner of uniting the ends of the strips to form a belt, and of fastening together the ends of the belt to make a continuous band for running over pulleys, is very important. A detailed discussion of these features is not necessary, however, in our present study of the subject.

66. Power of Belting. The amount of power which a given belt can transmit depends upon its speed, its strength, and its ability to adhere to the surface of the pulleys. The speed is usually assumed to be the same as the surface speed of the pulleys. The strength, of course, depends upon the width and thickness and upon the nature of the material of which the belt is made. The ability to cling to the pulley so as to run with little or no slipping depends upon the condition of the pulley surfaces and of the surface of the belt which is in contact with the pulleys and upon the tightness with which the belt is stretched over the pulleys. Since leather belts are more common and more nearly uniform in their character than those of other material, the discussion of power will be confined to them.

67. Unit of Power—Horse-Power. In order to measure the power, or the amount of work done, by any force, it is necessary to have some standard of measurement. A common unit for measuring work done is that known as the **foot-pound.** A foot-pound is the amount of work done in raising a one-pound weight a distance of one foot, or in moving any number of pounds through such a distance that the product of the force exerted multiplied by the distance moved is equal to one. For example, if a 12-lb. weight is lifted one-twelfth of a foot, the work done is 12 lbs.$\times\frac{1}{12}$ ft.$=1$ ft.-pound. If the apparatus furnishing the force to raise this weight is such that it can raise it in one minute, the apparatus is said to be capable of doing one foot-pound of work per minute, or to have a power of one foot-pound per minute.

For measuring large quantities of power a larger unit

is used, known as a horse-power. One **horse-power** is equal to 33,000 ft.-lb. of work per minute. For example, an engine which is capable of doing one horse-power work is one which can move 1 lb. through a distance of 33,000 ft. per minute, or 33,000 lb. 1 ft. per minute, or any number of pounds through such a distance in a minute that the product of the force multiplied by the distance moved in a minute is 33,000.

68. Tension in a Belt. Effective Pull. In Fig. 54, suppose the pulley A is fast to the shaft S and the pulley B fast

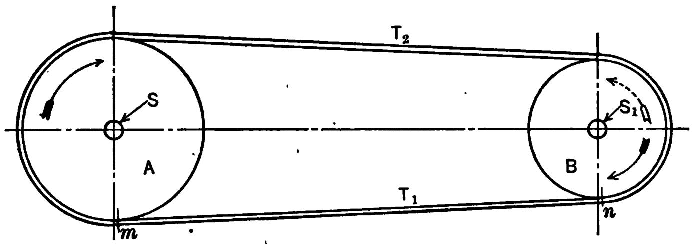


FIG. 54.

to the shaft S_1. Let it be assumed that when the shafts are at rest a belt 1 in. wide is stretched over the pulleys as shown, the tightness with which it is stretched being such that there is a tension or pull in the belt of a definite number of pounds. This tension is practically the same at all places in the belt and is called the *initial tension.* Let this initial tension be represented by the letter T_0. Suppose, now, that shaft S is caused to turn in the direction of the arrow by some external force. As it begins to turn the lower part of the belt, say from m to n, will become tighter, that is, will have a tension greater than T_0 lb. and the upper part will tend to slacken, or have a tension less than T_0 lb. Let this new tension in the lower or tight side be represented by T_1 and that in the upper or slack side by T_2. If the belt sticks to the pulley B so that there is no slipping, the force T_1 tends to cause the pulley B to turn as shown by the full arrow and

the force T_2 tends to cause B to turn as shown by the dotted arrow. As soon as T_1 becomes enough greater than T_2 to overcome whatever resistance the shaft S_1 offers to turning, the pulley B will begin to turn in the direction of the full arrow.

The unbalanced force, then, which makes the driven pulley B turn is the difference between the tension T_1 on the tight side of the belt and the tension T_2 on the slacker side of the belt. This difference in tensions is called the *effective pull* of the belt and is here represented by the letter E.

From the above discussion it may be seen that the following equation holds true:

$$T_1 - T_2 = E. \quad \quad . \; . \; . \; . \; . \; . \quad (21)$$

69. To Find the Horse-power of a Belt. Since, as explained in the previous paragraph, the effective pull is the force in the belt which enables it to do work, it follows that *the product of the effective pull multiplied by the speed of the belt in feet per minute will give the foot-pounds of work per minute that the belt performs, and this divided by* 33,000 *will give the horse-power which the belt transmits.* If N is the r.p.m. of S and D the diameter of pulley A (in feet) the following equation expresses the horse-power of the belt.

$$\frac{\text{Belt speed in ft. per minute} \times E}{33{,}000} = \text{H.P.} \quad . \; . \quad (22)$$

$$\therefore \quad \frac{\pi DN \times (T_1 - T_2)}{33{,}000} = \text{H.P.} \quad . \; . \; . \quad (23)$$

It is evident from the above that for a given belt speed the greater difference there is between T_1 and T_2 the more horse-power the belt transmits. Experience has shown that T_1 should not be more than $2\frac{1}{3} \times T_2$, to avoid slipping and that the greatest value of T_1 which it is wise to count on is about 75 lb. per inch of width for single belt and 150 lb. per inch of width for a double belt to avoid overstraining the belt.

From these figures,

Maximum T_1 per inch of width $=75$ lb. for single belt,

and 150 lb. for double belt, (24)

and

Maximum $T_1=\frac{7}{3}T_2$. . . . . (25)

$T_2=32$ lb. (nearly) for single belt and 64 lb. for double belt.

Maximum $T_1-T_2=75-32$ or 43 lb. for a single belt. (26)

Maximum $T_1-T_2=150-64=86$ lb. for a double belt. (27)

That is, the greatest effective pull per inch of width for a single belt is about 43 lb. and for a double belt 86 lb. These values multiplied by the width of any given belt in inches will give the maximum effective pull for that belt.

Substituting these values in Eq. (22),

$$\frac{\text{Belt speed in ft.p.m.} \times 43 \times \text{width of belt in inches}}{33{,}000} = \text{H.P.}$$

a single belt will transmit. . (28)

$$\frac{\text{Belt speed in ft.p.m.} \times 86 \times \text{width of belt in inches}}{33{,}000} = \text{H.P.}$$

a double belt will transmit. . (29)

In the case of belts running at high speed—say 2500 ft. per minute or more—the centrifugal force on the belt puts in an added tension and should be allowed for if the power of the belt is calculated by the above rule.

A simple and somewhat more conservative rule for calculating the power that a given belt will transmit is known as the **millwrights' rule** and has been determined largely by practical experience. This rule is as follows:

A single belt traveling 1000 *ft. per minute will transmit* 1 *H.P. per inch of width and a double belt traveling* 560 *ft. per minute w ll transmit* 1 *H.P.* From this rule,

$$\frac{\text{Belt speed in ft. per minute} \times \text{width of belt in inches}}{1000} = \text{H.P.}$$

a single belt will transmit. . . (30)

$$\frac{\text{Belt speed in ft. per minute} \times \text{width of belt in inches}}{560} = \text{H.P.}$$

a double belt will transmit. (31)

The above rules enable us to calculate the power which a known belt running at a known speed can safely be depended upon to transmit, or they enable us to calculate how wide a belt is necessary to transmit a given horse-power if the speed is known.

Example 8. A shaft carrying a 48-in. pulley runs at a speed of 180 r.p.m. An 8-in. double belt runs over the pulley and drives another shaft. To find the power that the belt can be expected to transmit without excessive strain.

Solution 1. Using formula (29),

$$\text{Belt speed in feet} = \frac{\pi 48}{12} \times 180 = 2262 \text{ ft. per minute.}$$

Then

$$\frac{2262 \times 86 \times 8}{33{,}000} = 47 \text{ H.P. (nearly).}$$

Solution 2. Using formula (30),

Belt speed as in solution 1 = 2262,

$$\frac{2262 \times 8}{560} = 32 \text{ H.P.}$$

It will be noticed that the two solutions given above give widely different answers, that from the millwrights' rule being nearly 33 per cent less than the other. This percentage difference would not be as great for higher speeds. Any such solution for a belt must be approximate, and merely furnishes a means of estimating the horse-power roughly. There is no doubt that the above belt, if in proper condition, would carry much more than even the 47 H.P.,

but the heavier the belt is loaded the more attention it will require and the shorter will be the life of the belt.

Example 9. A shaft running 200 r.p.m. is driven by a single belt on a 24-in. pulley. 15 H.P. is required. To find a suitable width of belt to use.

Solution 1. Using formula (28),

$$\text{Belt speed} = \frac{\pi 24}{12} \times 200 = 1257 \text{ ft. per minute.}$$

Then,

$$\frac{1257 \times 43 \times \text{width}}{33{,}000} = 15,$$

$$\text{width} = \frac{15 \times 33{,}000}{1257 \times 43} = 9 \text{ in. nearly.}$$

Solution 2. Using formula (30),

$$\frac{1257 \times \text{width}}{1000} = 15,$$

$$\therefore \quad \text{width} = \frac{15{,}000}{1257} = 12 \text{ in. nearly.}$$

Here again the millwrights' rule shows a wider belt necessary for a given horse-power.

Example 10. Two shafts A and B are to be connected by a 12-in. double belt carrying 72 H.P. A is the driving shaft, B is to run 180 r.p.m. To find the size of the pulleys on A and B.

First find the necessary belt speed using Eq. (31).

$$\frac{\text{Belt speed} \times 12}{560} = 72,$$

$$\therefore \quad \text{Belt speed} = \frac{72 \times 560}{12} = 3360 \text{ ft. per minute.}$$

Since B is to turn 180 r.p.m., if x = the diameter of the pulley on B then

$$\pi \times x 180 = 3360$$

or

$$x = \frac{3360}{180\pi} = 5.94 \text{ ft.} = 71.28 \text{ in.}$$

or, since pulleys of that size would not be made in fractional inches a 72-in. pulley would be used.

$$\frac{\text{Pulley on } A}{\text{Pulley on } B}=\frac{180}{240}=54 \text{ in.}$$

70. Formulas for Calculating the Length of Belts. In finding the length of belt required for a known pair of pulleys at a known distance apart, the most satisfactory method, when possible, is to stretch a steel tape over the actual pulleys after they are in position. Often, however, it is necessary to find the belt length from the drawings before the pulleys are in place or when, for some other reason, it is not convenient actually to measure the length. Various formulas have been devised by which the length may be calculated when the pulley diameters and distance between centers of the shafts are known. These formulas, if exact, are all more or less complex and are, of course, different for crossed and for open belts. If the distance between shafts is large, the following will give an approximate value for the length of the belt.

Length of belt = 1.57 × sum of diameters of pulleys + twice the distance between centers, . . (32)

or, expressing the same in terms of the letters shown in Fig. 55, letting L represent the belt length,

$$L=1.57(D+d)+2C. \qquad (33)$$

D, d, and C, must be expressed in like linear units, if in feet, the resulting value of L will be in feet; if in inches the value of L will be in inches.

In the case of an open belt where the two pulleys are of the same diameter the above formula gives an exact answer. If the pulleys are not of the same diameter, the length of belt obtained by Eq. (32) will be less than the correct length. If the shafts are several feet apart and the difference in diameters of the pulleys is not great, the percentage error is

very small for an open belt. With a crossed belt, pulleys of medium size, and the shafts several feet apart, the error from the use of Eq. (33) is very slightly less than the real length. These equations are accurate enough to use in estimating the length of belt.

The following equations, which are given in this place without proof, may be used to calculate the belt length very close to the exact value. Their principal use is for deriving equations for calculating stepped pulleys, as will be seen later.

Open Belt. Referring to Fig. 55, and letting L represent the belt length,

$$L=1.57(D+d)+2C+\frac{(D-d)^2}{4C}. \quad . \quad . \quad . \quad (34)$$

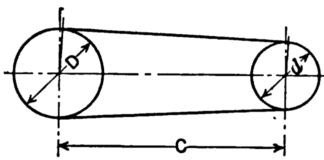


FIG. 55.

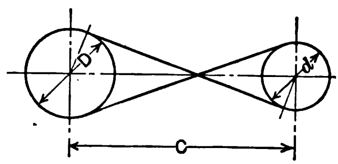


FIG. 56.

Crossed Belt. Referring to Fig. 56,

$$L=1.57(D+d)+\left(.017 \sin^{-1}\frac{(D+d)}{2C}\right)(D+d)$$
$$+\sqrt{4C^2-(D+d)^2}, \quad . \quad . \quad . \quad . \quad (35)$$

in which the term $\left(\sin^{-1}\frac{D+d}{2C}\right)$ means the number of degrees in the angle whose sine is $\frac{D+d}{2C}$.

Example 11. An open belt connects a 48-in. pulley with a 24-in. pulley. Distance between centers of the pulleys 6 ft. To find belt length. Use Eq. (34). First express the pulley diameters in feet namely, 4 ft. and 2 ft.

Then

$$L=1.57(4+2)+2\times6+\frac{(4-2)^2}{4\times6}=9.42+12+\frac{4}{24}=21.58 \text{ ft.}=21 \text{ ft. } 7 \text{ in.}$$

Example 12. To find the lengths of a crossed belt to run over the same pulleys described in Example 11, using Eq. (35). First look up in a table of trigonometric functions the angle whose sine is $\frac{4+2}{2\times6}$ or $\frac{1}{2}$. This is found to be 30°.

Then,

$$L=1.57\times(4+2)+.017\times30\times6+\sqrt{4\times6^2-2^2}=9.42+3.06+\sqrt{140}$$
$$=9.42+3.06+11.83=24.31 \text{ ft.}=24 \text{ ft. } 4 \text{ in.}$$

71. Stepped Pulleys. Sometimes it is necessary to have such a belt connection between two shafts that the speed of the driven shaft may be changed readily while the speed of the driving shaft remains constant. One method of accomplishing this is the use of a pair of pulleys each of which has several diameters as shown in Fig. 57. Such pulleys are known as *stepped pulleys*. Suppose that the shaft S, Fig. 57, is the driver, making N r.p.m. When the belt is in the position shown in full lines, the working diameter of the pulley A is D_1 and the working diameter of pulley B is d_1. Then if n_1 represents the r.p.m of S_1, when the belt is in this place we have $\frac{n_1}{N}=\frac{D_1}{d_1}$. (See § 62.)

If the belt is shifted to any other position, as that shown by dotted lines, D_x becomes the working diameter of the driving pulley and d_x of the driven pulley. If n_x represents the speed of S, for this belt position we have as before

$$\frac{n_x}{N}=\frac{D_x}{d_x}. \qquad (36)$$

Therefore, by properly proportioning the diameters of the different pairs of steps it is possible to get any desired series of speeds for the driven shaft.

In designing such a pair of pulleys two things must be taken into account. First, the ratio of the diameters of the

successive pairs of steps must be such as to give the desired speed ratios. Second, the sum of the diameters of any pair of steps must be such as to maintain the proper tightness

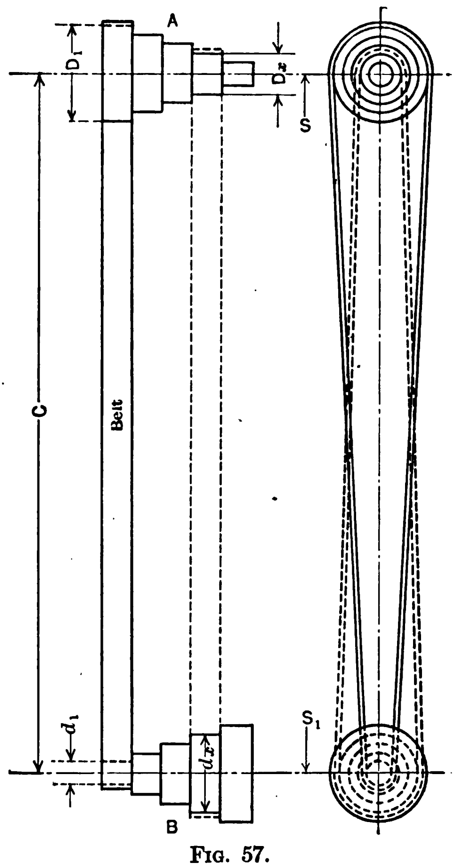


FIG. 57.

of the belt for all positions. This second consideration makes the problem of design considerably more complicated.

Two cases arise: First, the design of the pulleys for a crossed belt and second, the design for an open belt.

72. Stepped Pulleys for Crossed Belt. Assuming that the values of D_1, N, n_1, n_x, and C are known for the drive shown in Fig. 57, and assuming that the belt is crossed, instead of open as there shown, let it be required to find a method for calculating D_x and d_x.

First find d_1, which is readily done from the equation

$$\frac{n_1}{N}=\frac{D_1}{d_1},$$

in which d_1 is the only unknown quantity. Knowing, then, D_1 and d_1 the value of D_1+d_1 is known.

From Eq. (35) the length of the belt to go over the steps D_1 and d_1 is

$$1.57(D_1+d_1)+.017\sin^{-1}\frac{D_1+d_1}{2C}(D_1+d_1)+\sqrt{4C^2-(D_1+d_1)^2}.$$

When the belt is on the steps whose diameters are D_x and d_x, the equation for the length of belt is

$$1.57(D_x+d_x)+.017\sin^{-1}\frac{D_x+d_x}{2C}(D_x+d_x)+\sqrt{4C^2-(D_x+d_x)^2}.$$

Since the same belt is to be used on both pairs of steps, the value of these two equations must be the same.

Therefore,

$$\begin{aligned}1.57(D_x+d_x)+\left(.017\sin^{-1}\frac{D_x+d_x}{2C}\right)(D_x+d_x)+\sqrt{4C^2-(D_x+d_x)^2}\\=1.57(D_1+d_1)+\left(.017\sin^{-1}\frac{D_1+d_1}{2C}\right)(D_1+d_1)\\+\sqrt{4C^2-(D_1+d_1)^2}.\end{aligned}$$

An inspection of the above expression will show that if (D_x+d_x) is equal to (D_1+d_1) the two sides of the equation will be equal. That is,

$$D_x+d_x=D_1+d_1. \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Therefore in designing a pair of stepped pulleys for a crossed belt the sum of the diameters of all pairs of steps must be the same.

Then from Eq. (36),

$$\frac{n_x}{N}=\frac{D_x}{d_x},$$

and Eq. (37),

$$D_x+d_x=D_1+d_1,\ D_x \text{ and } d_x,$$

may be found by the method of simultaneous equations.

Example 13. To find the diameters of all the steps in the pulleys shown in Fig. 58 if a crossed belt is to be used.

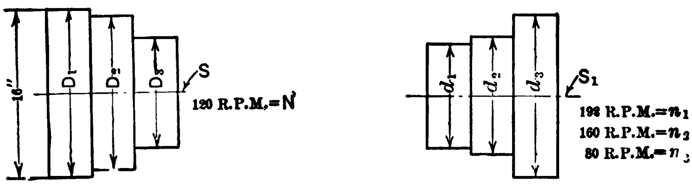


FIG. 58.

First, find d_1 from the equation $\frac{n_1}{N}=\frac{D_1}{d_1}$ or

$$\frac{192}{120}=\frac{16}{d_1}$$

whence

$$d_1=\frac{16\times 120}{192}=10 \text{ in.}$$

Therefore,

$$D_1+d_1=16+10=26 \text{ in.}$$

From Eq. (37)

$$D^2+d^2=D_1+d_1=26 \text{ in.}$$

and

$$\frac{D_2}{d_2}=\frac{160}{120},$$

or

$$D_2=\tfrac{4}{3}d_2.$$

Substituting this value of D_2 in the preceding equation,

$$\tfrac{4}{3}d_2+d_2=26,$$

or

$$\tfrac{7}{3}d_2=26,$$

whence

$$d_2=\frac{26\times 3}{7}=11\tfrac{1}{7}\text{ in.},$$

and

$$D_2=26-11\tfrac{1}{7}=14\tfrac{6}{7}.$$

Again,

$$D_3+d_3=26\text{ in.},$$

and

$$\frac{D_3}{d_3}=\frac{80}{120},$$

or

$$D_3=\tfrac{2}{3}d_3,$$

$$\therefore\quad \tfrac{2}{3}d_3+d_3=26,$$

or

$$\tfrac{5}{3}d_3=26\text{ in.},$$

whence

$$d_3=15\tfrac{3}{5}\text{ in.},$$

and

$$D_3=26-15\tfrac{3}{5}=10\tfrac{2}{5}\text{ in.}$$

73. Stepped Pulleys for Open Belt. Referring to Fig. 57 again assume D_1, N, n_1, n_x and C to be known and let the belt be open; to find a method for calculating D_x and d_x.

First find d_1 as in § 71, from the equation $\frac{n_1}{N}=\frac{D_1}{d_1}$.

The length of the belt when on the first pair of steps is given by the expression in Eq. (34).

$$1.57(D_1+d_1)+2C+\frac{(D_1-d_1)^2}{4C}.$$

When the belt is shifted to the steps whose diameters are D_x and d_x, the expression for the length is

$$1.57(D_x+d_x)+2C+\frac{(D_x-d_x)^2}{4C}.$$

Therefore,

$$1.57(D_x+d_x)+2C+\frac{(D_x-d_x)^2}{4C}$$

$$=1.57(D_1+d_1)+2C+\frac{(D_1-d_1)^2}{4C}.$$

Solving this equation for the quantity

$$(D_x+d_x),$$

$$(D_x+d_x)=(D_1+d_1)+\frac{(D_1-d_1)^2-(D_x-d_x)^2}{6.28C}. \quad . \quad (38)$$

This equation with the equation

$$\frac{n_x}{N}=\frac{D_x}{d_x},$$

might be solved by the method of simultaneous equations to find the values of D_x and d_x. This method is rather long, however, and it is customary to find values for D_x and d_x by the method given for a crossed belt. Then from these values find a value for D_x-d_x and assume that the value is near enough to the correct value to substitute in Eq. (38) to get the value of $(D_x-d_x)^2$. Eq. (38) then becomes simple and may be solved simultaneously with $\frac{n_x}{N}=\frac{D_x}{d_x}$ to find values of D_x and d_x which are practically correct.

Example 14. To find the diameters of all the steps in the pulleys shown in Fig. 59, if an open belt is to be used.

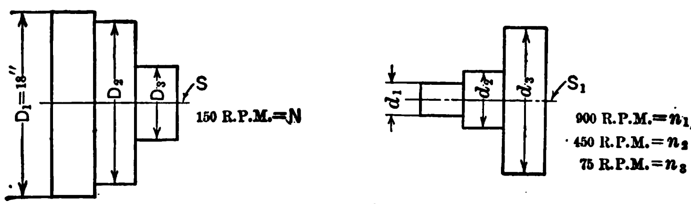


FIG. 59.

First find d_1 from the equation

$$\frac{n_1}{N}=\frac{D_1}{d_1},$$

or

$$\frac{900}{150}=\frac{18}{d_1},$$

whence

$$d_1 = \frac{150 \times 18}{900} = 3 \text{ in.}$$

Therefore,

$$D_1 + d_1 = 18 \text{ in.} + 3 \text{ in.} = 21 \text{ in.}$$

To solve for D_2 and d_2, $D_2 + d_2$ must first be found from Eq. (38), using the approximation explained in connection with that equation That is, first find a value for D_2 and d_2 assuming that

$$D_2 + d_2 = D_1 + d_1 = 21 \text{ in.}$$

Call these values D'_2 and d'_2

$$\frac{n_2}{N} = \frac{D'_2}{d'_2}$$

or

$$\frac{450}{150} = \frac{D'_2}{d'_2},$$

whence

$$D'_2 = 3d'_2.$$

Therefore,

$$3d'_2 + d'_2 = 21 \text{ in.},$$

or

$$d'_2 = 5.25,$$

and

$$D'_2 = 21 - 5.25 = 15.75 \text{ in.}$$

Then

$$D'_2 - d'_2 = 15.75 - 5.25 = 10.50,$$

and

$$(D'_2 - d'_2)^2 = (10.5)^2 = 110.25.$$

Assuming that this value is nearly enough correct to use in place of $(D_x - d_x)$ Eq. (38), we have,

$$D_2 + d_2 = 21 + \frac{225 - 110.25}{6.28 \times 24}$$

$$= 21 + .76 = 21.76 \text{ in. nearly.}$$

Then from this equation $D_2 + d_2 = 21.76$,

and

$$\frac{D_2}{d_2} = \frac{450}{150},$$

or

$$D_2 = 3d_2,$$

$$3d_2 + d_2 = 21.76.$$

Therefore,

$$d_2 = 5.44,$$

and

$$D_2 = 16.32 \text{ in.}$$

In a similar manner assume $D_3 + d_3 = 21$ and solve for approximate values D'_3 and d'_3 getting $D'_3 = 7$ in. and $d'_3 = 14$ in.

Whence

$$(D'_3 - d'_3)^2 = 7^2 = 49.$$

Then

$$D_3 + d_3 = 21 + \frac{225 - 49}{6.28 \times 24} = 21 + 1.17 = 22.17,$$

$$\frac{D_3}{d_3} = \frac{75}{150} = \tfrac{1}{2}.$$

Therefore

$$D_3 = 7.39$$

and

$$d_3 = 14.78 \text{ in.}$$

74. Corrections Necessary. The proportion chosen in the data for Example 14 gives an extreme case, and it will be noticed that even here the amount that $D_2 + d_2$ varies from $D_1 + d_1$ is only about $\frac{3}{4}$ in. and the variation of $D_3 + d_3$ is a trifle less than $1\frac{3}{16}$ in. These quantities are large enough to affect the tightness of the belt and must, therefore, be taken into account. In ordinary cases, however, where the distance between centers is much larger than in Example 14 and where the speed ratios are not so great the value of $D_x + d_x$, as obtained from Eq. (38) by the method just illustrated differs but very little from $D_1 + d_1$ and this difference can usually be neglected.

75. Equal Stepped Pulleys. It is common practice, when convenient, to design a pair of stepped pulleys in such a way that both pulleys have the same dimensions and can, therefore, be cast from the same pattern. This condition imposes

certain restrictions on the speed ratios as may be seen from the following:

Referring to Fig. 60, if the pulleys are alike,

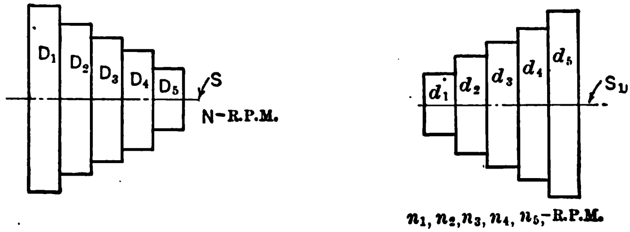


FIG. 60.

$$D_1=d_5,\ D_2=d_4,\ D_3=d_3,\ D_4=d_2,\ D_5=d_1.$$

As in previous discussions

$$\frac{n_1}{N}=\frac{D_1}{d_1},$$

and

$$\frac{n_5}{N}=\frac{D_5}{d_5},$$

but

$$\frac{D_5}{d_5}=\frac{d_1}{D_1}.$$

Therefore,

$$\frac{n_1}{N}=\frac{N}{n_5}. \qquad (39)$$

In a similar manner,

$$\frac{n_2}{N}=\frac{N}{n_4}, \qquad (40)$$

and

$$N=n_3. \qquad (41)$$

That is: *When equal stepped pulleys are used the speeds of the driven shaft must be so chosen that the speed of the driving shaft is a mean proportional between the speeds of the driven shaft for belt positions symmetrically either side of the middle step.* . . . . (42)

Example 15. A pair of equal three-stepped pulleys, Fig. 61, are to connect two shafts. The driving shaft makes 120 r.p.m., and the

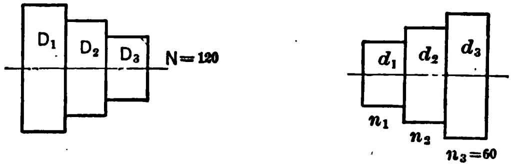


FIG. 61.

lowest speed of the driven shaft is 60 r.p.m. To find the other two speeds of the driven shaft.

By formula (42),

$$\frac{n_1}{N}=\frac{N}{n_3},$$

or

$$\frac{n_1}{120}=\frac{120}{60}.$$

Therefore

$$n_1=240,$$

$$n_2=N=120.$$

If the step diameters are to be calculated, it will be done by the methods explained in §72 or §73 according as the belt is crossed or open.

76. Speed Cones. In some cases instead of stepped pulleys, pulleys which are approximately frusta of cones are used, as shown in Fig. 62. Here the working diameters of the pulleys, as D_x and d_x for any belt position, are measured at the middle of the belt. To design such a pair of pulleys a series of diameters D_1, D_2, D_3, etc. (Fig. 63), may be calculated in the same way as steps and plotted at equal distances (a) apart, then a smooth line drawn through their ends, as shown. The length (a) does not affect the problem except as it makes the cone longer or shorter. For open belts the pulleys would not be straight lines (see Fig. 64).

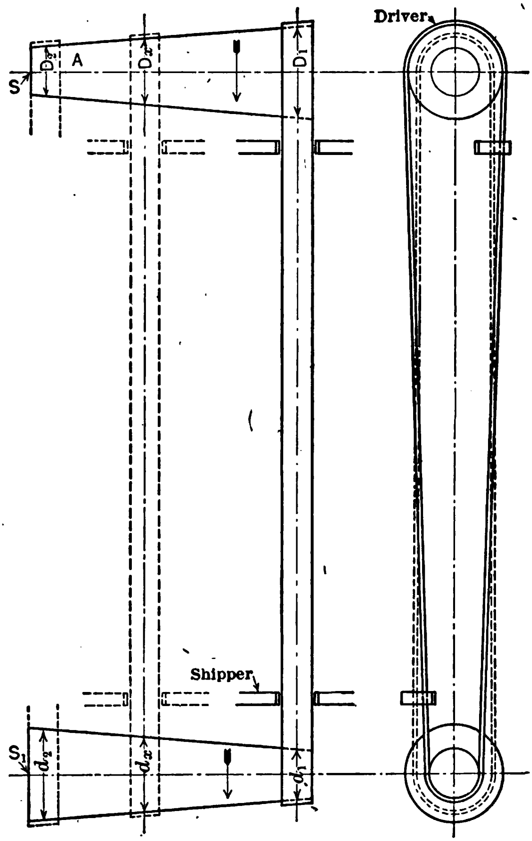


FIG. 62.

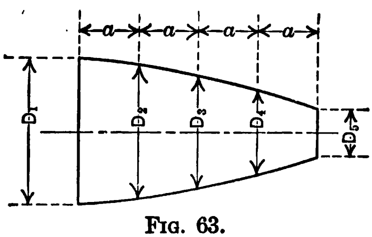


FIG. 63.

When cones are used, a *shipper* must guide each part of the belt just at the point where it runs on to the pulley

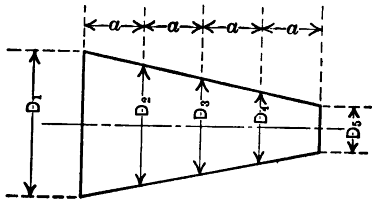


FIG. 64.

(see Fig. 62); otherwise the belt will tend to climb toward the large end of each pulley. Both shippers must be moved simultaneously when the belt is shifted.

77. Belt Connections between Shafts which are not Parallel. Non-parallel shafts may be connected by a flat belt with satisfactory results, provided the pulleys are so located as to conform to a fundamental principle which governs the running of all belts, namely: *The point where the pitch line of the belt leaves one pulley must lie in a plane which is perpendicular to the axis of the pulley toward which it is running and if produced will pass through the center of the face of the pulley toward which it is running.* This may be seen by a reference to Fig. 65. In this case the shafts S and T are intended to turn in the directions indicated by the arrows. Considering Elevation A, the pitch line of the belt leaves the pulley M at the point a. If the pulley N is in such a position on the shaft T that a plane through the middle of its face contains the point a, the belt will run properly on to pulley N. XX is the trace of this plane and evidently contains point a. Similarly, in Elevation B, the pitch line of the belt leaves the pulley N at b_1 and M is so located on shaft S that a plane YY through the middle of its face contains b_1. In other words, each pulley is in line with the delivering face of the other pulley.

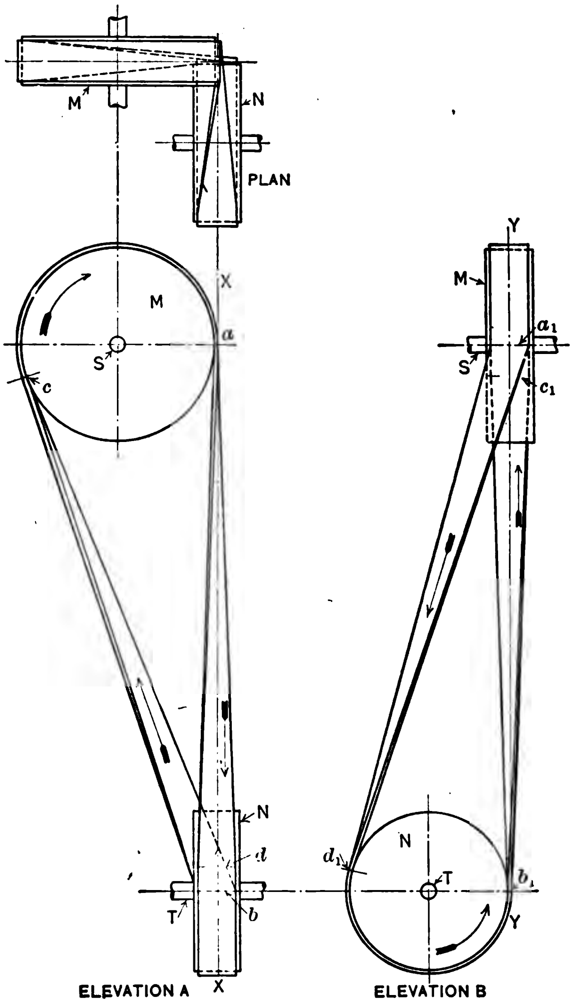

M
N
PLAN
Y
X
M
M
a
a_1
S
S
c
c_1
N
d
d_1
N
T
T
b
b_1
Y
ELEVATION A
X
ELEVATION B

FIG. 65.

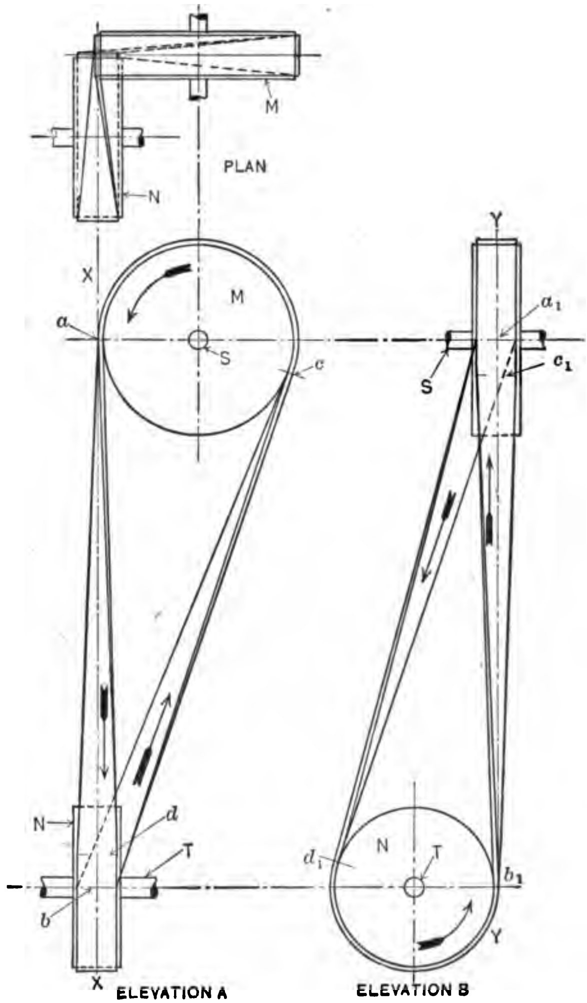


FIG. 66.

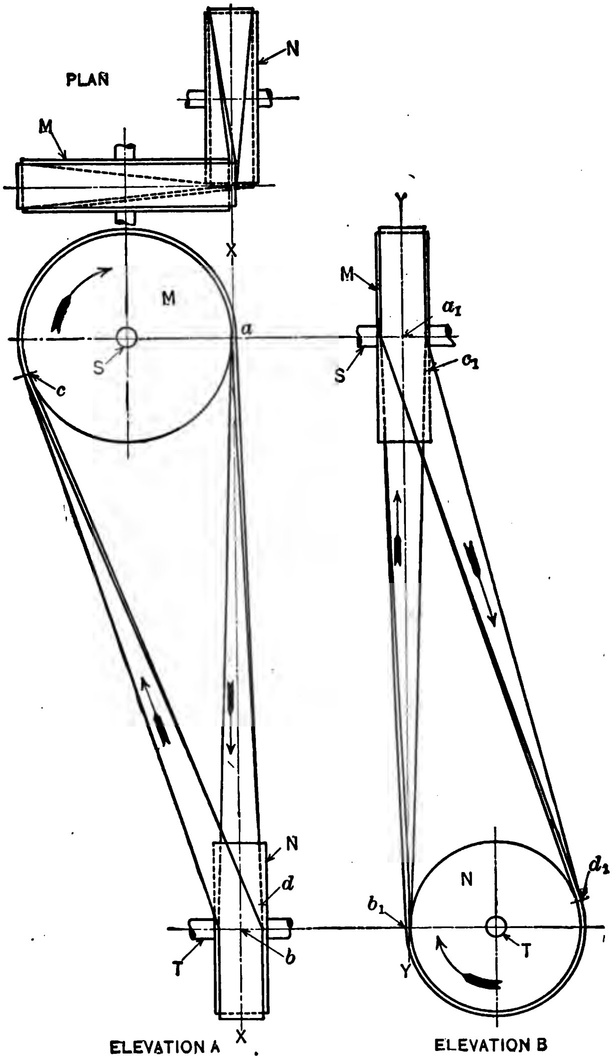

PLAN
N
M
X
M
a
S
c
Y
M
a_1
S
c_1
N
d
b
T
X
ELEVATION A
N
d_1
b_1
T
Y
ELEVATION B

FIG. 67.

Fig. 66 shows the proper relative position of the pulleys if the direction of turning of shaft S is the reverse of that in Fig. 65; Fig. 67 shows the pulleys with the direction of shaft T reversed, S turning as in Fig. 65. Fig. 68 shows the pulleys with both shafts reversed in direction from Fig. 65. The various points are lettered to correspond in the four figures.

78. Quarter-turn Belt. A belt which connects two non-intersecting shafts at right angles with each other, similar to those in Figs. 65 to 68, is called a *quarter-turn* belt. Emphasis should be laid on the fact that, for any given setting of the pulleys, the shafts must always turn in the direction in which they were designed to turn. If the direction of rotation is changed without resetting the pulleys, the belt will immediately leave the pulleys. For this reason simple quarter turn belts like those illustrated above are likely to give trouble if used in places where there is possibility of the shafting turning backwards even a small fraction of a turn. If this should happen to a small belt, it could easily be replaced on the pulleys; in the case of a large belt, however, the replacing would be more difficult.

79. Reversible Direction Belt Connection between Non-parallel Shafts—Guide Pulleys. If the connection between two non-parallel shafts is to be such that the shafting may run in either direction and still have the pulleys deliver the belt properly, in accordance with the fundamental law already explained, it is usually necessary to make use of intermediate pulleys to guide the belt into the proper plane. Such pulleys are called *guide pulleys*.

80. Examples of Belt Drives—Method of Laying Out. The following examples will illustrate a few of the types of belt drives which may occur and will give some idea of the method of procedure in designing such drives. Some of these examples are chosen from existing drives, others have been modified in order to illustrate the principles more clearly.

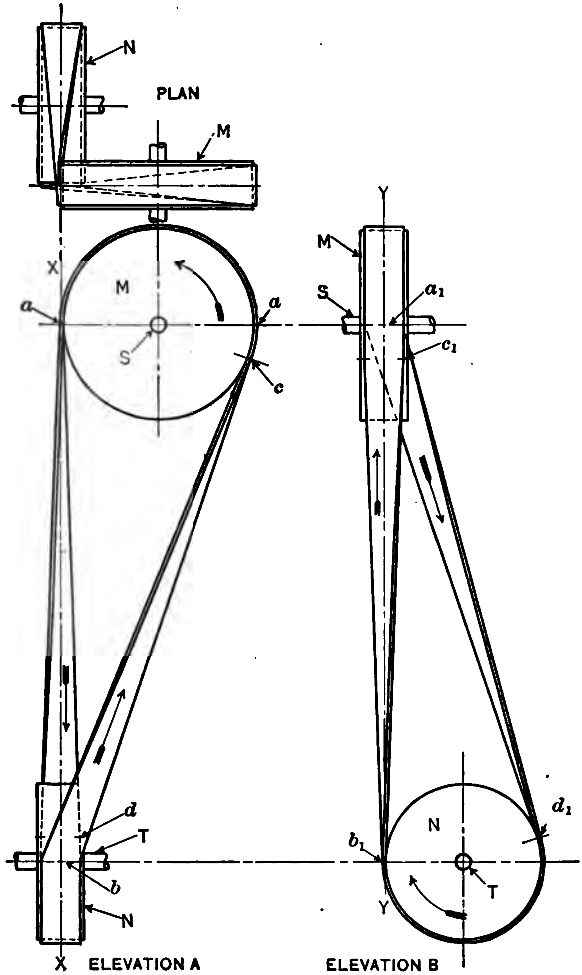

PLAN
N
M
X
M
S
a
a
c
d
T
b
N
X
ELEVATION A
Y
M
S
a_1
c_1
N
b_1
d_1
T
Y
ELEVATION B

Fig. 68.

Example 16. Two shafts are located as shown in Fig. 69, and turn in directions indicated by arrows. A 30-in. pulley on shaft S is to drive a 24-in. pulley on shaft T by means of a 6-in. double belt. The drive is to be a simple quarter turn without guide pulleys, and, therefore, capable of turning only in one direction. Holes 3×10 in. are to be cut in the floor in the proper positions to allow the belt to run through them centrally. To draw two elevations and a plan of the pulleys and belt and show the holes in the floor.

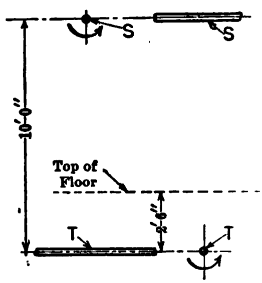


Fig. 69.

Solution. (See Fig. 70.) First, draw two horizontal center lines GG_1 and KK_1 at a distance apart equal to the distance between the shafts (in this case 10 ft.). At a convenient point on the upper center line choose S for the center of the upper shaft and, with S as a center, draw a circle whose diameter is equal to the diameter of the pulley on S (30 in.). Put on the arrow to indicate the direction of rotation. Tangent to this circle on the side which is moving downward draw a vertical line X_1X. With X_1X and KK_1 as center lines draw the rectangle 1–2–3–4, whose length is equal to the diameter of the pulley on T and whose width is equal to the width of face of this pulley. At any convenient place on KK_1, to the right, select the point T for the center of the end view of the lower shaft and with T as a center draw the circle whose diameter is equal to the diameter of the lower pulley, putting on the arrow to indicate the direction of rotation. Tangent to this circle on the side which is moving upward draw the vertical line YY_1 and with YY_1 and GG_1 as center lines draw the rectangle 5–6–7–8 which forms the side view or right elevation of the upper pulley. We now have the two elevations of the pulleys properly placed to turn in the required directions in conformity to the

law stated in § 77. The plan may be above either elevation and is drawn by projecting up from the two elevations

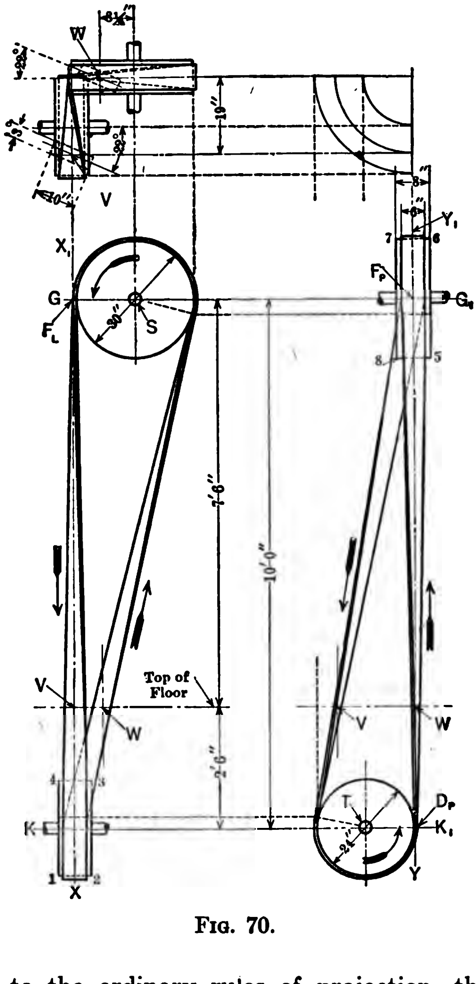


FIG. 70.

according to the ordinary rules of projection, the process being suggested in the drawing by the dotted projection lines and arcs. The true projections of the edges of the belt

would be lines slightly curved, but they are ordinarily represented by straight lines.

To find the position of the belt holes, project from the elevations to find the position of the points where the pitch line of the belt pierces the plane of the floor. These points are V and W and are the center points of the belt holes. Since the belt twists through an angle of 90° in a distance of 10 ft. it will twist through a proportionate amount in passing from the pulley to the floor. Considering first the part of the belt which is moving downward, the following would be the equation for the angle which the hole at V makes with the upper shaft:

$$\frac{\text{Angle with upper shaft}}{90°}=\frac{7'-6'}{10'}.$$

Therefore the angle with the upper shaft is 68° about, or 22° with lower shaft. Then, through V draw a line at 22° with the axis of the lower shaft and on this as a center line construct a rectangle 3×10 in., which is the plan view of the hole. The other hole is found in the same way.

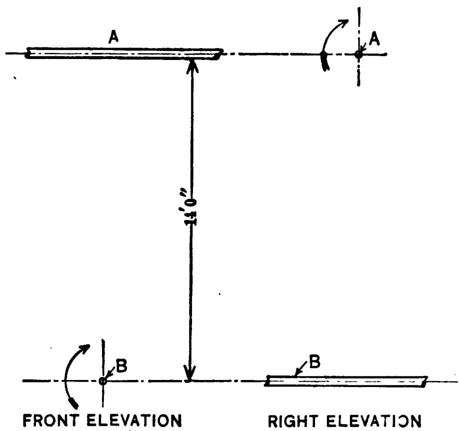


FIG. 71.

Example 17. Given two shafts A and B located as shown in Fig. 71. Shaft A, carrying a 52-in. pulley, is to drive a 60-in. pulley on shaft B by means of a 12-in. double belt. Two guide pulleys 30 in. diameter in line with each other are to be located on two horizontal shafts so that the

direction of rotation may be reversed without the belt running off. When turning in the direction indicated by the arrows, the tight side of the belt is to run direct from the driven to the driving pulley in a vertical line, the loose side returning around the guide pulleys. The guide pulley which receives the belt from the upper main pulley is to be on a shaft whose center is 9 ft. below the center of A; the other guide pulley is to be on a shaft whose center is 2 ft. 6 in. below the center of A. Main pulleys 14 in. face, guide pulleys 12-in. face.

To draw two elevations and a plan.

Solution. Referring to Fig. 72, first draw the center line YY_1 which is the center line through the shaft B. At a distance of 14 ft. above YY_1 draw XX_1 as the center line of shaft A. At any convenient place near the left end of YY_1 choose the point B_L as the center point of shaft B for the left elevation. With B_L as a center draw a circle 60-in. diameter which will be the left elevation of the pulley on B. Next, draw the vertical line TT_1 tangent to this pulley on the side which is moving upward. TT_1 is the pitch line of the tight part of the belt and must be contained in the center plane of the pulley on A. With TT_1 and XX_1 as center lines, draw the rectangle 1–2–3–4 of length equal to the diameter of the pulley on A and width equal to the width of face of the same pulley. This rectangle is the side view or left elevation of the upper or driving pulley. Next, choose a point A_P near the right end of XX_1 and draw the end view or right elevation of this pulley. The vertical line $T_{1P}T_P$ drawn tangent to the upward moving or driving side of this pulley will be the right elevation of the driving or tight part of the belt and must be contained in the center plane of the driven pulley, since the direction of rotation is to be reversible. Therefore, the lines $T_{1P}T_P$ and YY_1 are the center lines for the rectangle 5–6–7–8 which is the right elevation of the driven pulley, TT_1 and T_PT_{1P} are the two views of the line of intersection of the center planes of the pulleys.

We have now determined the position of the two main pulleys with respect to each other and have drawn the two

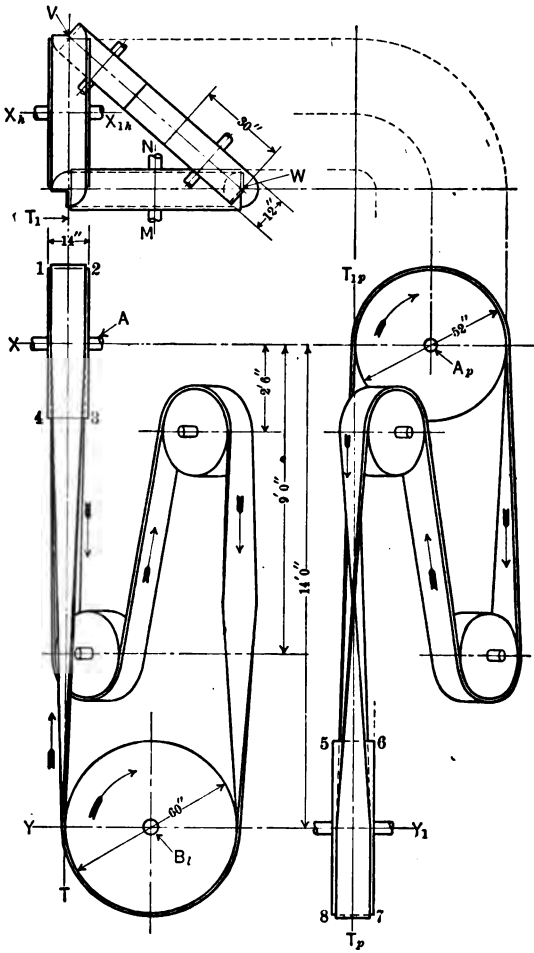


FIG. 72.

elevations; their place must next be drawn. This may be placed above either elevation, and is here placed above the

left elevation. Looking down on the pulleys, both will appear as rectangles. The center line X_hX_{1h} may be drawn at any convenient distance above XX_1 and is the horizontal projection, or plan view, of the center line of the shaft A. The pulley on this center line can be projected directly up from the rectangle 1–2–3–4 and will, of course, have the same dimensions. The center line MN of the shaft B will be vertically above B_L and the rectangle which forms the plan view of the pulley on B will be located on this center line with its middle line passing through the front end of the plan of the other pulley. In other words, the plan view of these two pulleys is obtained by projecting from the two elevations in accordance with the ordinary principles of projections.

To draw the guide pulleys, first locate them in the plan. Their center plane will contain the line VW and one will have its contour passing through point V in plan while the contour of the other will pass through W. To draw the elevations, first draw the center lines at the specified distances below shaft A and then draw the ellipses which represent the pulleys by projecting from the plan.

Example 18. Shaft S (Fig. 73) drives shaft T by means of an 8-in. double belt. Both main pulleys 36-in. diameter located as shown. The usual direction of rotation to be as indicated by the arrows but the arrangement to be such that the directions may be reversed. Two 15-in. guide pulleys are to be placed on a vertical shaft to carry the belt between the two main pulleys. All pulleys 9-in. face.

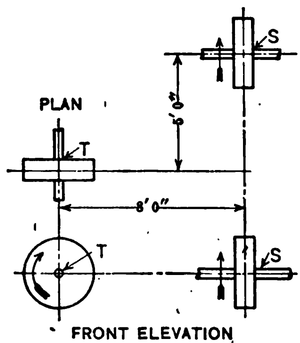


FIG. 73.

To draw the drive, making the elevations and a plan.

Solution. (See Fig. 74.) Draw the three views of the

main shafts and pulleys, the plan and front elevation being the same as shown in Fig. 73, and the right elevation being constructed from these in accordance with the usual prin-

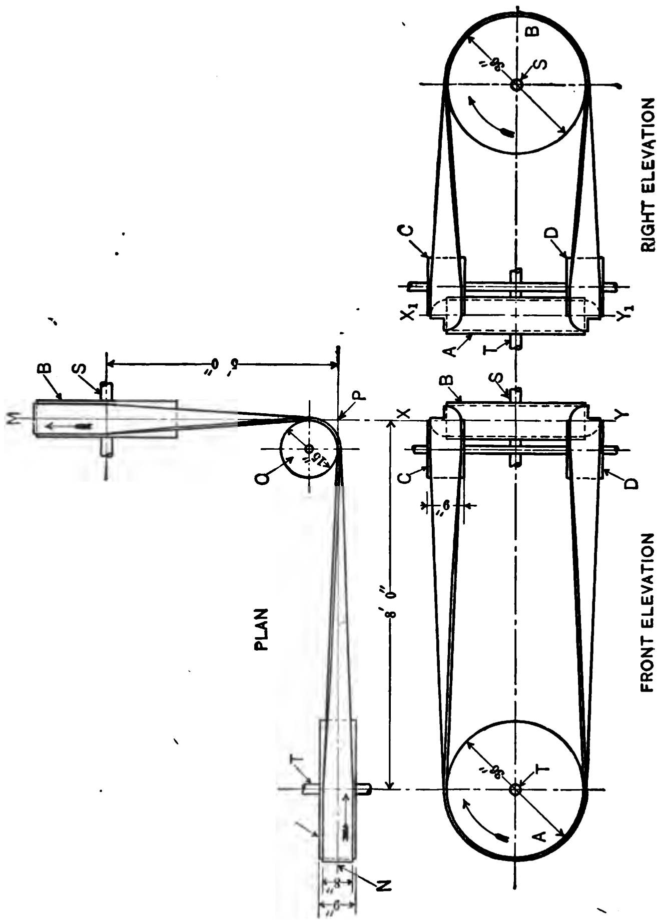


Fig. 74.

ciples of projection. The left elevation might have been made instead of the right.

The position of the guide pulley shaft can best be deter-

mined from the plan. The planes of the pulleys A and B intersect in a line which, in plan, is projected as the point P and in the two elevations as the lines XY and X_1Y_1, respectively. Since the upper guide pulley is to deliver the belt to pulley B, it must be tangent to the line PM, and since, if the direction of rotation is reversed, C must be able to deliver the belt to pulley A it must be tangent to the line PN (see § 77). The same reasoning will apply to the lower guide pulley. The center of the guide pulley shaft will, therefore, be at a point which is distant from PM and PN an amount equal to the radius of the guide pulleys. Since the pulleys A and B are of the same diameter and their axes on the same level, the guide pulley C will appear in the elevations with its center plane tangent to the tops of A and B and D will have its center plane tangent to the bottoms of A and B. With this arrangement it is possible for either of the main pulleys to deliver the belt into the plane of either guide pulley, and either guide pulley may deliver to either main pulley.

It should be noticed that a drive like this, with both guides on the same vertical shaft, can be reversible in direction only when the main pulleys are of the same diameter. The next two examples show the construction when the main pulleys are of different diameters.

Example 19. Referring again to Fig. 73, assume the same conditions as for Example 18, except that the main pulleys are of different diameters. Suppose the pulley on T is 48-in. diameter and that on S 36-in. diameter. The direction of rotation not to be capable of being reversed.

Solution. See Fig. 75. The three views of the main shafts and pulleys are drawn as in Example 18. The center of the guide pulley shaft is located in the plan at such a point that the pulley circumference will be tangent to the lines PM and PN as in Fig. 74. The position of the guide pulley C on this shaft is determined in the front elevation, it being at such a height that its center plane will be tangent to the top surface (that is, contain the point of delivery K)

of the pulley A which delivers the belt to C. Similarly, in the right elevation, the position of the guide pulley D is

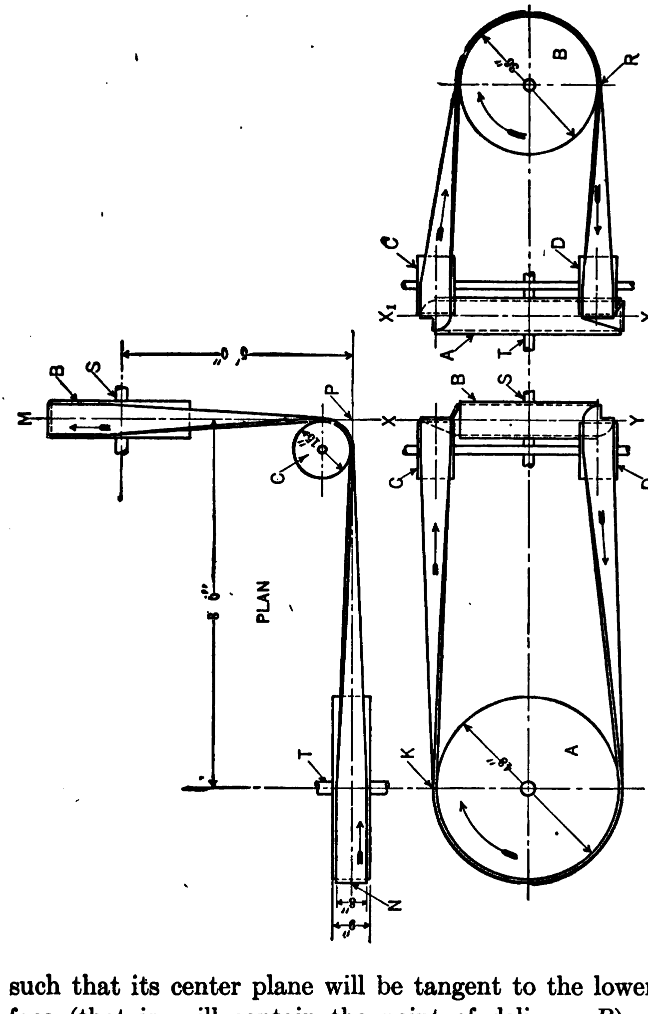


FIG. 75.

such that its center plane will be tangent to the lower surface (that is, will contain the point of delivery R) of the pulley B.

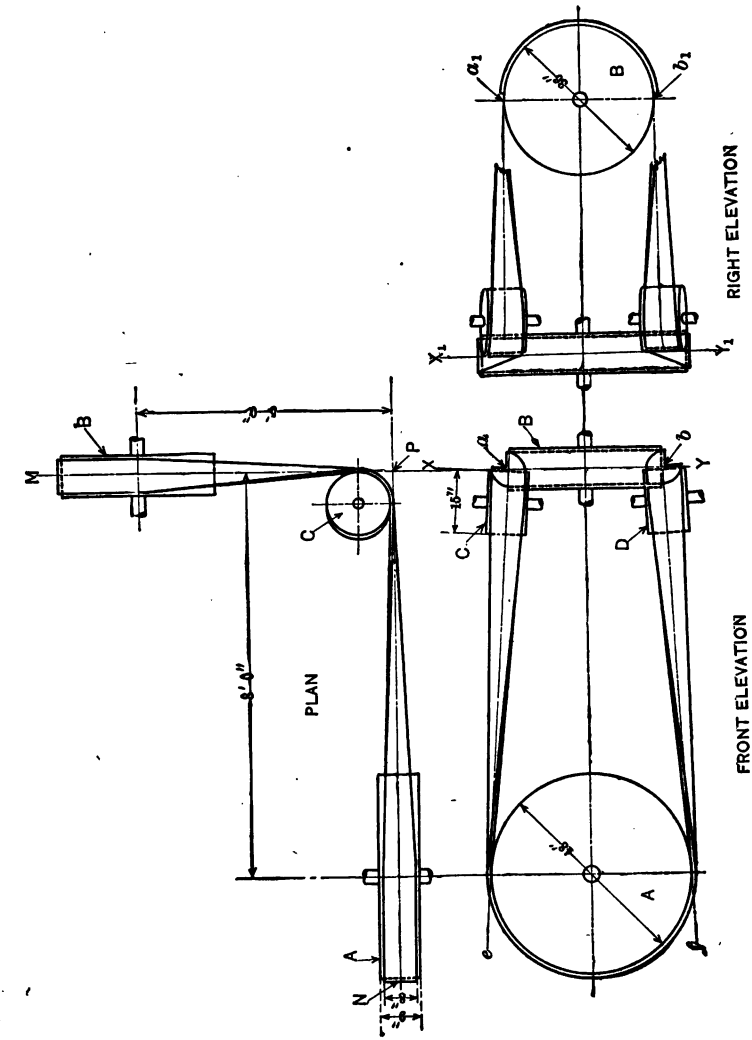


FIG. 76.

Example 20. With the data the same as for Example 19, suppose it is required so to arrange the guide pulleys that the direction of rotation may be reversed. (They cannot in this case be on the same vertical shaft.)

Solution. See Fig. 76. After having drawn the three views of the main shafts and pulleys, the problem becomes one of so placing the guide pulleys that they will conduct the belt in either direction. There are a great many possible solutions of this problem, but that shown in Fig. 76 is the simplest.

In the front elevation the points *a* and *b* are the center points of the upper and lower contour elements of the pulley *B*. From *a* and *b* draw lines *ae* and *bf* tangent to pulley *A*. The center planes of the guide pulley *C* must contain the line *ae* and the center plane of the guide pulley *D* must contain the line *bf*. *C* will appear in this view, therefore, as a rectangle with one end passing through *a* and *D* will appear as a rectangle with one end passing through *b*. In the other views the edges of the guide pulleys will appear as ellipses, as shown.

Example 21. The shaft *S*, Fig. 77, is to drive the shaft

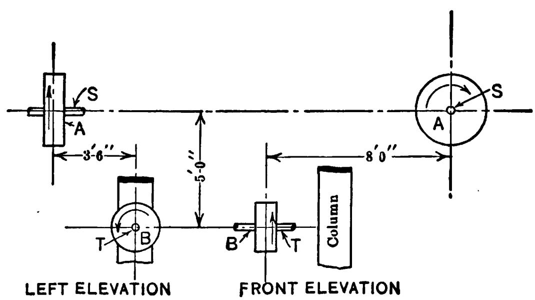


FIG. 77.

T by means of an 8-in. belt running on pulleys *A* and *B*. One of the columns of the building makes it impossible to

continue the shaft T far enough to place the pulley B in the proper position relative to A to permit the use of a direct

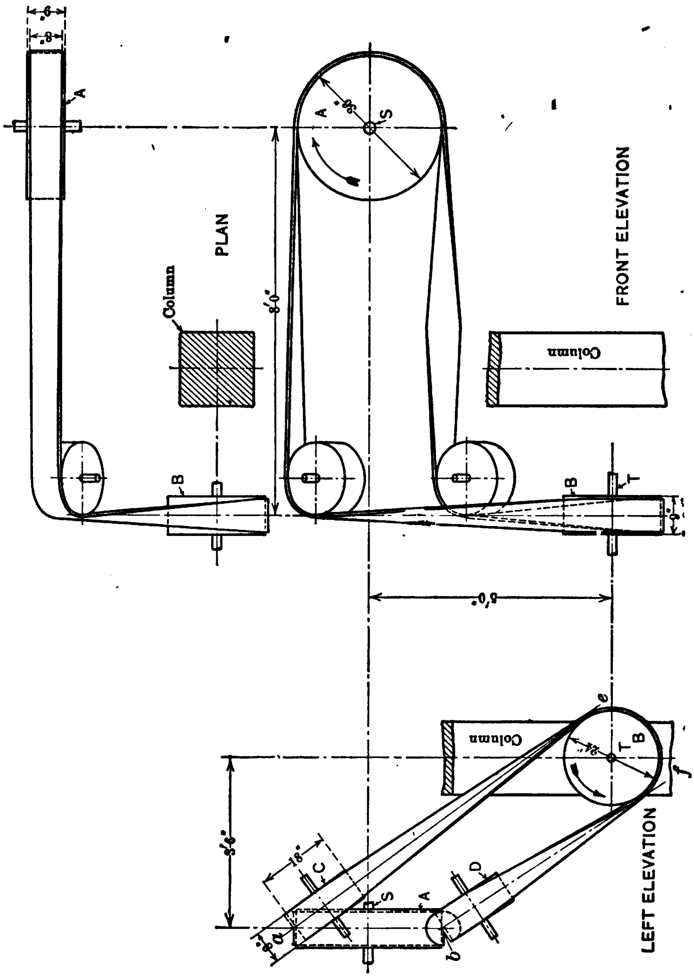


FIG. 78.

quarter-turn drive. Furthermore the vertical distance between the two shafts is too small to make such a drive

practicable even if the column did not interfere. It is, therefore, necessary to employ guide pulleys to conduct the belt from B to A and from A back to B. The relative directions of rotation are to be as shown and the guide pulleys are to be so located that the directions may be reversed. 18-in. guide pulleys will be used.

Solution. See Fig. 78. The positions of the guide pulleys are determined from the left elevation. From the center points a and b of the upper and lower contour lines of pulley A, lines ae and bf are drawn tangent to the pulley B. The center plane of one guide pulley C must contain the line ae and the pulley will appear in this view as a rectangle with one end passing through a. The center plane of the other guide pulley D must contain the line bf and the pulley will appear as a rectangle with one end passing through b. The guide pulleys will appear in the front elevation and in the plan with their edges ellipses, as shown. The two guide pulleys so nearly coincide in the plan that the lower one was omitted in the drawing.

It will be noticed that in all the preceding examples the same surface of the belt comes in contact with the main pulleys at all times. This is an important condition from a practical standpoint. Whenever possible the same surface should run against the guide pulleys also.

Example 22. Given two shafts at right angles, located as shown in Fig. 79. Shaft A carries a 52-in. pulley which drives a 60-in. pulley on shaft B by means of a double belt 12 in. wide. The ordinary direction of rotation is as shown by the arrows. One guide pulley 30-in. diameter is to be so located that the direction of rotation may be reversed without the belt running off. When turning in the direction shown, the tight side of the belt is to run direct from driven to driving pulley in a vertical line, the loose side returning around the guide pulley.

The main pulleys are 14 in. wide and the guide pulley 12 in. wide. Two elevations and a plan are to be drawn.

Solution. It will be noticed that, except for the guide pulley, this problem is the same as Example 17 and the method of drawing the three views of the main pulleys is exactly as described for that case.

To draw the guide pulley proceed as follows: (See Fig. 80.) The distance of this guide from either one of the main pulleys would be governed somewhat by convenience in actually

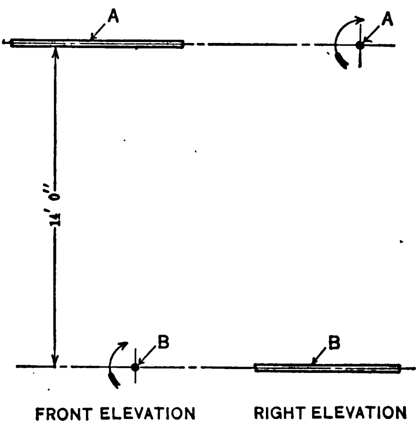


Fig. 79.

setting up the bearings to support it, and partly also by the relative sizes of the main pulleys. It is desirable so to locate it as to give the least possible abruptness to the bend in the belt. In this case there has been selected a point C in the line of intersection of the two main pulley planes which is 6 ft. 6 in. below the axis of the upper shaft. This point will be at C_L and C_P in the two elevations. From C_L draw a line tangent to the lower pulley at D_L and project across, getting the other view of this point at D_P. In a similar way draw a line from C_P tangent to the upper pulley at E_P and project across to find E_L. We now have the two projections of two lines CD and CE drawn from a point in the inter-

section of the pulley planes tangent to the two pulleys, and the guide pulley must be set in such a position that its center plane will contain these two lines. The problem then is to draw the projections of the guide pulley when so set. Either elevation may be drawn first; let us start with the left elevation. Here C_LD_L shows in its true length and the line CD itself may be considered as lying in the plane of the paper. The line CE has one end C_L in the plane of the paper while the line itself really slants down below the paper. The true size of the angle which it makes with the plane of the paper is equal to the angle $E_PC_PF_P$. D_LC_L is the trace, or line of intersection with the paper, of the plane containing CD and CE.

Now swing this plane up into the paper to get the true angle between CD and CE. To do this produce D_LC_L to the left and through E_L draw a line perpendicular to D_LC_L. From C_L with a radius equal to the true length of CE (that is, with radius C_PE_P) cut this perpendicular at E_1 and join E_1 to C_L. The angle $E_1C_LD_L$ is the true angle between the lines EC and DC. Set the compasses with a radius equal to the radius of the guide pulley and find by trial the center O_1 about which a circle of this radius may be drawn tangent to C_LE_1 and C_LD_L. This point shows the real position of the center of the middle circle of the guide pulley relative to the lines CE and CD. The next step is to revolve O_1 back to find its projection relative to C_LE_L and C_LD_L. To do this draw a line through E_1 and O_1 meeting C_LD_L at H_1; then join H_1 to E_L. Through O_1 draw a line perpendicular to D_LC_L meeting H_1E_L at O_L and C_LD_L at J_L. Then O_L is the projection of the center point of the pulley. To draw the projections of the edges of the pulley select any point as J_2 in the line C_LD_L, draw a line through O_L parallel to C_LD_L and from J_2 with a radius equal to the radius of the guide pulley cut this line at O_2. Draw a line through J_2 and O_2, also a line V_2W_2 through O_2 perpendicular to J_2O_2. On these two lines as center lines

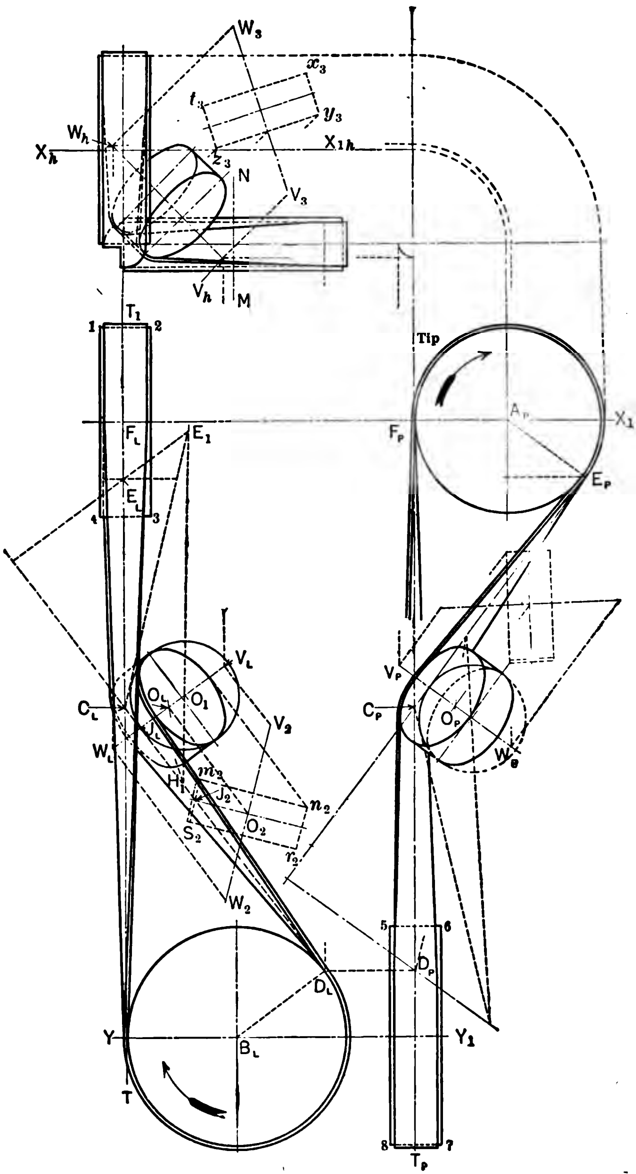

W3
x3
t
y3
Wh
Xh
X1h
z3
N
V3
Vh
M
T1
1
2
Tip
FL
E1
AP
X1
FP
EP
EL
4
3
VL
VP
CL
OL
O1
V2
CP
OP
JL
WL
W8
H
m2
J2
n2
S2
O2
r2
W2
5
6
DP
DL
Y
BL
Y1
T
8
7
TP

Fig. 80.

construct the rectangle $m_2n_2r_2s_2$ of length equal to the diameter of the guide pulley and width equal to the width of face of the same. This rectangle is the side view of the guide pulley when revolved up into the plane of the paper, the line V_2W_2 being the center line of its shaft. The projection of the pulley consists of two equal ellipses with major axes equal to the diameter of the guide pulley and minor axes found by projecting the points m_2n_2 and s_2r_2 on to the line O_LJ_L, as shown. If a definite length V_2W_2 is chosen for the shaft, the projections of the ends of the shaft are found at V_L and W_L. The right elevation of the guide pulley and the axis of its shaft are found by a method exactly similar to that just described.

The plan view of the guide pulley is found by first finding the plan projections V_h and W_h of the two ends of the shaft by projecting up from the two elevations, then revolving this line V_hW_h over until it comes into the plane of the paper, as shown at V_3W_3. On this line is drawn the rectangle $t_3x_3y_3z_3$, representing the pulley, and the axes of the ellipses which constitute the plan view of the pulley are found by projecting the rectangle $t_3x_3y_3z_3$ on to V_hW_h, as shown.

81. Rope Driving. The transmission of power by means of ropes did not come into general use until a comparatively few years ago. At the present time, however, ropes have, in many instances, taken the place of belts and gears for heavy drives in grain elevators, steel mills, and textile mills.

The following discussion consists largely of condensed extracts from *The Blue Book of Rope Transmission*, published by the American Manufacturing Company, of Brooklyn, N. Y., and the illustrations are reproduced from that publication.

Some of the advantages claimed for rope drives are as follows:

1. *The Distance and Direction in which Power may be Transmitted is Practically Unlimited.* Satisfactory driving

may be done where the distance beween shafts is as great as 175 ft., without the aid of carrying pulleys, while, with such carriers, the distance may be prolonged almost indefinitely. On the other hand, successful driving can be done with ropes where the shafts are close together. There are in operation many drives whose sheaves are but 10 ft., or even less, center to center. Where shafts are neither in the same line nor plane, by properly placed guide sheaves power may be transmitted around corners from one building to another, and, in short, between any two shafts in whatever relative position they may be placed.

2. *The Amount of Power which may be Transmitted is also Practically Unlimited.* There are many drives in this country which are transmitting from 3000 to 4000 H.P. with perfect satisfaction.

3. *Economy in First Cost and Maintenance.* In drives of 200 H.P. and up, and where the shafts are more than 30 ft. center to center, the cost as compared with belts will vary from 15 to 50 per cent in favor of ropes, according to the distance and size of drive. This advantage increases rapidly as the distance apart of shafts and amount of power transmitted increase.

The small cost of maintenance of a rope drive is a strong point in its favor. The average life of rope on a properly designed drive is from eight to ten years.

4. *Economy of Space.* The width of rim surface required in rope drives is only from one-half to two-thirds that required for belting, varying with the size of rope used. It follows, therefore, that the supporting bearings may be placed nearer together for a rope than for a belt drive.

5. *Positive and Steady Running.* The elasticity of the rope, its light weight, and slackness between pulleys take up inequalities in power and load.

6. *Rope Drives are Noiseless.* This fact is due to the flexibility and lubrication of the rope, and the air passage in the groove, between it and the sheaves.

7. *No Electrical Disturbance is Produced.* This is an advantage particularly noted in textile mills, where such disturbances, caused by belts, are the greatest source of annoyance.

8. *Precise Alignment of Shafting not Necessary.* When shafts are at an angle with each other, in the same or a different plane, by properly placed guide pulleys power may be transmitted by ropes to great advantage. And where shafting supposed to be parallel has, for one reason or another, been placed at an angle, unless the imperfection is too pronounced, the rope will follow the grooves of the sheave, even without the use of an idler.

9. *No Loss by Slipping.* In properly designed rope drives, where diameters of the pulleys are sufficiently large and the angle of groove correctly turned, loss of power by slipping becomes so infinitesimal that no allowance should be made when calculating the speed for the driven shaft.

10. *Transmitting Power to Different Floors.* Where power is to be carried to several floors of a mill, rope-driving again stands forth prominently. The full number of ropes start from the driving pulley, while the number required for each different shaft are easily dropped off at the several floors.

11. *Future Additions to Power.* It has become in this country almost a universal custom, in the erection of new mills, to provide for increase of plant. In rope-driving this provision is readily made by installing sheaves with extra grooves, and adding new ropes when additional power is required.

82. Systems of Rope Driving. There are two distinct systems of rope-driving, the *multiple,* or, as it is popularly known, *English* system, and the *continuous,* or *American* system. Each of these has its advantages, depending upon the conditions under which it is used.

The Multiple System (Fig. 81) is the simpler, consisting of one or more independent ropes running side by side in

the grooves of the pulleys. This system is especially adapted to the transmission of large powers, and gives the very best results for drives protected from the weather, where the shafts are parallel or very nearly so. With this system, the drive has the utmost security against breakdowns, because of the extreme unlikelihood of more than one rope giving

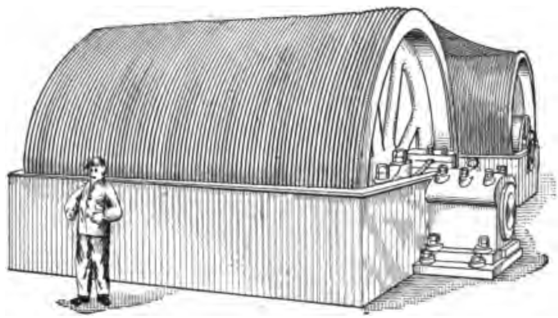


FIG. 81.

way at the same time. When a failure does occur, the individual rope may be removed and repaired at some convenient opportunity, the delay from such failure being slight, if any. Further, power may be more easily carried to the different floors of a mill; the amount of power transmitted may be more readily increased by the addition of new ropes; the rope always bending in the same direction has a longer lease of life than the Continuous System; and finally, it is cheaper to install.

In the *Continuous System* (Figs. 82 and 83) one rope is wound around the driving and driven sheaves several times. With this system it is necessary by some device to conduct the rope from an outside groove of the delivering, to the opposite outside groove of the receiving pulley, this transfer being accomplished by means of a traveling tension carriage whose office is to produce a uniform tension throughout the rope, and is so arranged as to travel back and forth,

automatically regulating the slack which may occur from stretch in rope, inequalities of load, etc.

The principle involved in the care of tension is important,

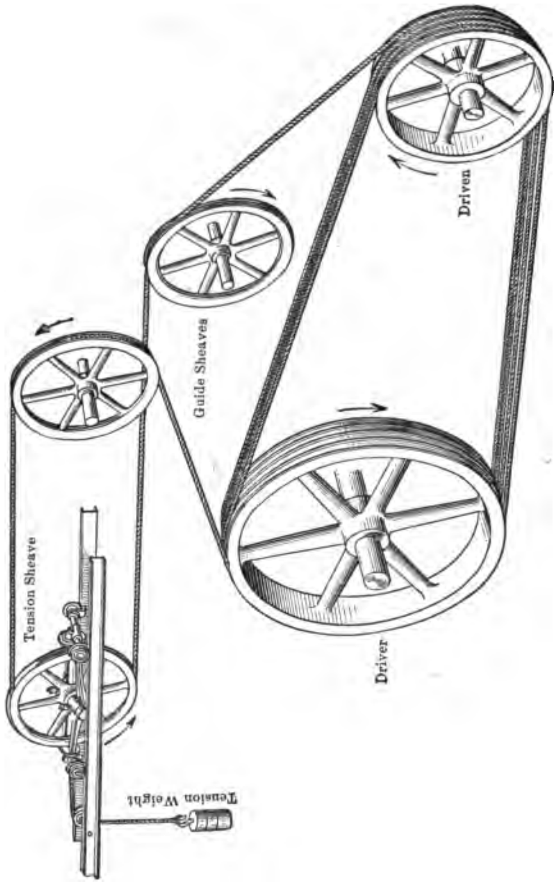


FIG. 82.

and when installing continuous drives every precaution must be taken that it is not violated. *The slack should be taken care of where it accumulates,* which is on the slack side of drive just off the driving sheave. There are two general

ways of accomplishing this: In the first of these, the rope is conducted from an outside groove of the driver to the tension sheave, and after passing around it, is returned to the opposite outside groove of the driven sheave. (Fig. 82.)

Secondly, where it is inconvenient to take slack directly from the driver, the same result may be obtained by taking it from the driven, the rope being led from an outside groove —which is a loose or independent sheave—to the tension sheave, and thence returned to the opposite outside groove

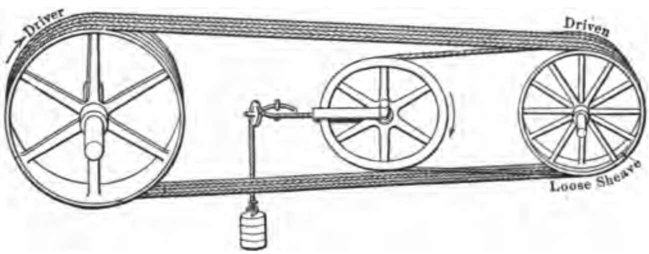


FIG. 83.

of the driven sheave. (Fig. 83.) This variety is known as a *self-contained drive.* These two methods admit of almost unlimited variations. Figs. 84, 85 and 86 show further examples.

The Continuous System is the more versatile. It is especially adapted to vertical and quarter-turn drives, where shafts are at an angle to each other, where rope is exposed to the weather, and in any special case of complicated transmission.

It is often erroneously claimed that ropes of the same diameter will exert more horse-power with the Continuous than with the Multiple System. This idea has probably arisen from the fact that makers of the Continuous System, as a general rule, require their ropes to do more work than is the custom with designers of the Multiple System. As a rope will do just so much work in a given time, the prac-

tice of increasing the horse-power by adding to the tension weights is a bad one, for, while it may increase the adhesive

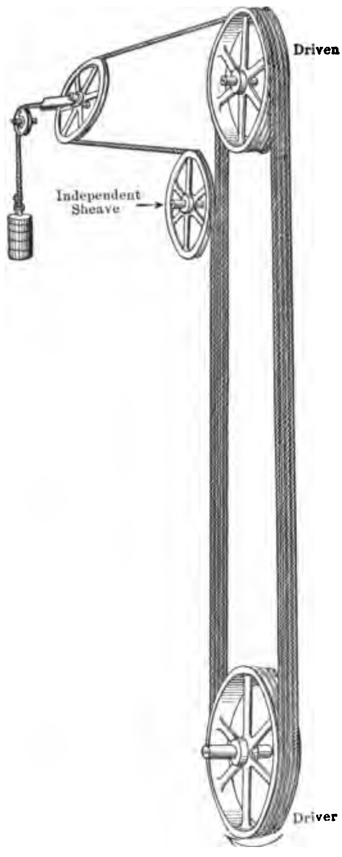


FIG. 84.

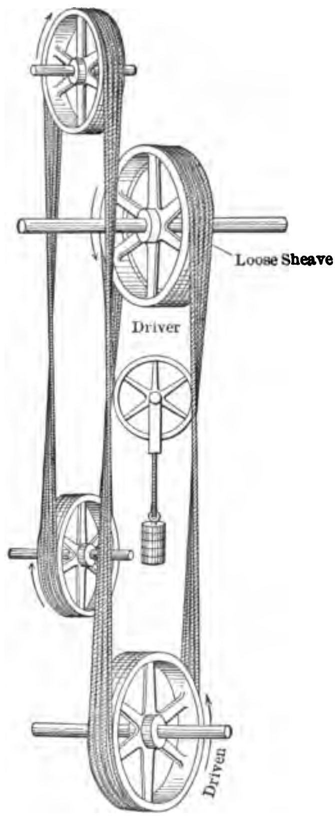


FIG. 85.

power, the extra strain put upon the rope necessarily shortens its life.

There is no definite rule which can govern the amount of tension weight necessary, as it varies with length of rope,

diameter of sheaves, amount of work, etc., but to secure best results and longest life of rope this weight should be just sufficient to give absolute security against any tendency on the part of the rope to slip.

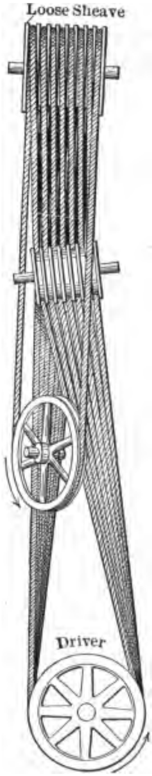


FIG. 86.

83. Rope Sheaves and Grooves. In rope drives the pulleys, or sheaves, are second in importance only to the rope itself. The two prominent factors in the construction of the rope sheave are, the relative proportion of its diameter to the diameter of rope used, and the shape and finish of groove. Transmission ropes wear first internally, the inner yarns becoming disintegrated by the friction caused from constant and rapid bending around sheaves. To limit this disintegration as far as possible, the diameter of the sheave should be made as large as practicable; experience has shown that for economical results a rope should not be made to bend about a sheave whose diameter is less than thirty-six times that of the rope, and *where space and speed of shaft will permit, forty diameters should be used as the minimum.* The use of large sheaves not only adds to the life of a rope, but the larger surface of contact gained increases its working power.

An unlimited variety of grooves have been tried, with more or less success, from which two general designs have evolved themselves, one for the Continuous and one for the Multiple System, though in their essential feature, i.e., angle of groove, they are similar. All driving and driven sheaves should be turned with a wedge or V-shaped groove of such size and depth that the rope used can never touch the bottom. Experience has

shown that, for general work, the best results are obtained with an angle of 45°.

Except in unusual cases an angle sharper than 45° is not advisable, while experience has proved that the flatter angle, 50 or 60°, permits the rope to slip slightly when subjected to varying loads or to an overstrain.

For idler or carrier sheaves either the V- or U-shaped grooves may be employed. Where the drive runs outside

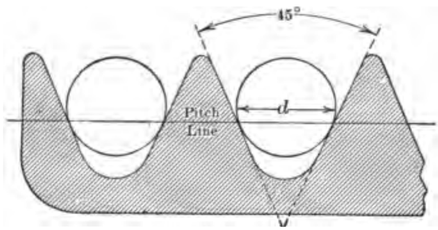


FIG. 87.

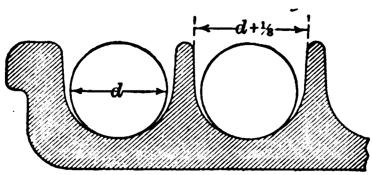


FIG. 88.

and the rope is exposed to the wind, the deep U groove should be used.

As in the Continuous System the rope is kept taut, it is not so apt to jump from the groove which, consequently, can be made more shallow than for the Multiple System; in fact, many drives are now running where the rope extends beyond the flanges of the grooves.

Figs. 87 and 88 illustrate the form of working and idle grooves generally adopted for the Continuous System.

Fig. 89 shows-cross sections of working grooves for different diameter ropes as used for the Multiple System, and frequently for the Continuous. This form is styled *Engineers' Standard Groove.* It will be noted in Fig. 89 that the

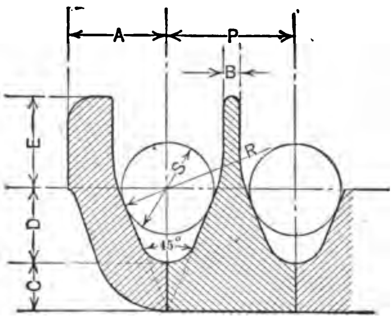


FIG. 89.

All Dimensions in Inches.

Size S	P	A	B	C	D	E	R
$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{15}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$2\frac{15}{16}$
$\frac{7}{8}$	$1\frac{1}{2}$	$1\frac{1}{16}$	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	$2\frac{15}{16}$
1	$1\frac{7}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{13}{16}$	1	$3\frac{7}{8}$
$1\frac{1}{8}$	2	$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{9}{16}$	$\frac{15}{16}$	$1\frac{1}{8}$	$3\frac{7}{8}$
$1\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{8}$	1	$1\frac{1}{4}$	4
$1\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{9}{16}$	$\frac{3}{8}$	$\frac{11}{16}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$3\frac{1}{8}$
$1\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{11}{16}$	$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$3\frac{5}{16}$
$1\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{15}{16}$	$\frac{3}{8}$	$\frac{7}{8}$	$1\frac{7}{16}$	$1\frac{3}{4}$	$3\frac{3}{8}$
2	$2\frac{3}{4}$	$2\frac{1}{8}$	$\frac{3}{8}$	1	$1\frac{5}{8}$	2	$3\frac{9}{16}$

sides of groove at the pitch diameter are straight, while those of Fig. 90 are gently curved throughout, being arcs of circles, having radii proportioned to the diameter of the rope used. The sides are thus curved to assure the rope revolving in the groove, a condition sought by the practical mill man, as it causes uniform wear, thereby adding to the life of the

rope. In the Continuous System, owing to the nature of its construction, the rope necessarily revolves in the groove.

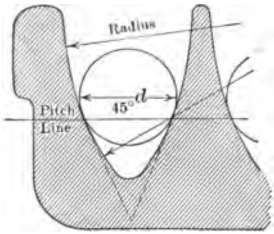


FIG. 90.

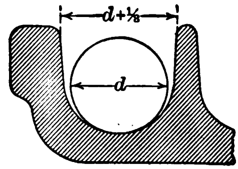


FIG. 91.

Fig. 91 is the usual form of groove adopted for idlers and carriers with the Multiple System. It differs from Fig. 88 only in that the groove is deeper, and the flanges, consequently, higher.

84. Data on Manila Transmission Rope. Table 3 furnishes convenient information concerning Manila rope.

TABLE III.—DATA ON MANILA TRANSMISSION ROPE

Diameter of Rope.	Square of Diam.	Approx. Shipping Wgt. per ft.	Breaking Strength.	Maximum Allowable Tension.	Length of Splice Feet. 3 Strands.	4 Strands.	6 Strands.	Smallest Diam. of Sheaves, Inches.	Maximum Number of Revolutions per Minute.
$\frac{3}{4}$	.5625	.21	3,950	112	6	8	..	28	760
$\frac{7}{8}$	.7656	.27	5,400	153	6	8	..	32	650
1	1.	.36	7,000	200	7	10	14	36	570
$1\frac{1}{8}$	1.2656	.45	8,900	253	7	10	16	40	510
$1\frac{1}{4}$	1.5625	.56	10,900	312	7	10	16	46	460
$1\frac{3}{8}$	1.8906	.68	13,200	378	8	12	16	50	415
$1\frac{1}{2}$	2.25	.80	15,700	450	8	12	18	54	380
$1\frac{5}{8}$	2.6406	.92	18,500	528	8	12	18	60	344
$1\frac{3}{4}$	3.0625	1.08	21,400	612	8	12	18	64	330
2	4.	1.40	28,000	800	9	14	20	72	290
$2\frac{1}{4}$	5.0625	1.80	35,400	1,012	9	14	20	82	255
$2\frac{1}{2}$	6.25	2.20	43,700	1,250	10	16	22	90	230

85. Power of Manila Rope Drives. The following table shows approximately the power that single Manila ropes will transmit under average conditions.

TABLE IV.—HORSE-POWER OF MANILA ROPE

Diameter of Rope.	Velocity, Feet per Minute.										
	1000	1500	2000	2500	3000	3500	4000	4500	5000	5500	6000
$\frac{3}{4}$	2.3	3.3	4.3	5.2	6.0	6.6	7.2	7.3	7.4	7.3	6.9
$\frac{7}{8}$	3.0	4.5	5.9	7.0	8.2	9.0	9.6	9.8	10.0	9.6	9.0
1	4.0	5.9	7.7	9.2	10.6	11.8	12.7	12.9	13.0	12.7	12.0
$1\frac{1}{8}$	5.0	7.5	9.7	11.6	13.5	14.9	16.0	16.3	16.7	16.5	15.3
$1\frac{1}{4}$	6.3	9.1	12.0	14.3	16.7	18.5	20.0	20.2	20.7	20.1	18.9
$1\frac{3}{8}$	7.5	10.8	14.4	17.4	20.0	22.1	23.7	24.5	24.6	24.0	22.3
$1\frac{1}{2}$	9.0	13.5	17.4	20.7	23.0	26.3	28.7	29.0	29.5	28.6	26.7
$1\frac{5}{8}$	10.5	15.5	20.1	24.3	27.9	30.8	32.9	34.1	34.3	33.3	31.0
$1\frac{3}{4}$	12.3	18.0	23.6	28.2	32.7	36.4	38.5	39.4	40.5	38.7	36.0
2	16.0	23.2	30.6	36.8	42.5	46.7	50.0	51.7	52.8	50.6	47.3
$2\frac{1}{4}$	20.0	29.6	38.6	46.6	53.6	59.2	63.6	65.8	66.3	64.4	60.3
$2\frac{1}{2}$	25.0	36.6	47.7	57.5	66.0	71.2	78.0	80.0	81.0	79.0	73.8

86. Size and Speed of Ropes. When designing a drive, it is well first to ascertain the largest sheave which may be used advantageously, and then adopt for the drive the largest rope possible, or, in other words, the rope whose diameter does not exceed one-fortieth of the diameter of the smallest sheave. For ordinary cases, however, it is not advantageous to use a rope larger than $1\frac{3}{4}$ in. in diameter; the 2-in. diameter rope and over should be used only in transmissions of great magnitude.

The most economical speed for a rope has been found from general practice to be from 4000 to 4500 ft. per minute, for, while the amount of work of which a rope is capable increases with the velocity, up to the rate of about 5200 ft.

per minute, it is equally true that the wear also increases directly with the speed, because the greater the velocity, the more bending and surface wear the rope must sustain.

87. Cotton Ropes. Cotton ropes are advantageously used as bands or cords on the smaller machine appliances; the fiber being softer and more flexible than Manila hemp, gives good results for small sheaves. For large drives, where power transmitted is in considerable amounts, cotton rope, as compared with Manila, is hardly to be considered, on account of the following disadvantages: It is less durable; it is injuriously affected by the weather, so that for exposed drives, paper-mill work, or use in water-wheel pits, it is absolutely unsatisfactory; it is difficult, if not impossible, to splice uniformly; even the best quality cotton rope is much inferior to Manila in strength, the breaking strain of the highest grade being but $4000 \times \text{diameter}^2$ as against $7000 \times \text{diameter}^2$ for Manila; while, for the transmission of equal powers, the cost of a cotton rope varies from one-third to one-half more than Manila.

88. Wire Ropes. Wire rope is well adapted for the transmission of large powers to great distances, as for instance in cable and inclined railways. Its rigidness, great weight, and rapid destruction due to bending, however, unfit it for use in mill service, where the average speed of rope is about 4000 ft. per minute. As the easiest way to break wire is by bending it, ropes made of it, by any method whatsoever, have proved unsatisfactory for drives of short centers and high velocity.

89. Chain Driving. The transmission of power by means of specially constructed chains running over suitably formed wheels or sprockets has many advantages over belt or rope drives, under certain conditions. When the shafts which are to be connected are so near together that a belt cannot be used to advantage and yet too far apart to connect by gears, the chain furnishes a very satisfactory means of connection. Furthermore, since the sprocket has teeth fitting into the

corresponding spaces in the chain, there can be no slipping and the connection is, therefore, as positive as if geared. No initial tension is required, that is, when the wheels are at rest there is no tension in the chain except that due to the weight of the chain itself. When running, the only tension (neglecting centrifugal force) is that due to the power which is being transmitted. This makes the pressure on the bearings less and there is, therefore, less wear and less friction loss. Since the effective pull is equal to the tension on the tight or driving side minus the tension on the slack side, and since the tension on the slack side is zero, the effective pull may be equal to the allowable working strength of the chain, whereas with a belt the effective pull can only be equal to the working strength of the belt minus the tension on the slack side necessary to prevent slipping.

One usually thinks of chain drives as being confined to low speeds. Chains, however, have stood up well when running at a linear speed as high as 4000 ft. per minute on sprockets making about 1200 r.p.m. In general it may be said that the larger the sprockets, up to a reasonable limit, the more satisfactory will be the drive. While sprockets having as few as 11 or 12 teeth may work fairly well under favorable conditions, it is not desirable to use a sprocket having less than 16 teeth. The ratio of the numbers of teeth in a pair of sprockets connected by a chain may vary from $\frac{1}{5}$ to $\frac{1}{8}$, depending upon the style of chain. The distance between centers of a pair of sprockets should not be less than $1\frac{1}{2}$ times the diameter of the larger sprocket nor more than 10 or 12 ft.

CHAPTER VI

INCLINED PLANE, WEDGE, SCREW, WORM AND WHEEL

90. Classification. Of the elements to be considered in this chapter the inclined plane and wedge come under the head of sliders, in division III of § 13, the screw and nut belong in division V, although it may involve straight sliding as well. The worm and wheel is an application of the screw principle combined with rotation of a wheel about a fixed axis.

91. Inclined Plane and Wedge. The inclined plane and wedge will be considered only as mechanical elements for producing motion or exerting force. In this sense they are essentially the same thing. In Fig. 92, P represents a wedge, or solid, whose lower surface mn is horizontal, resting on a horizontal surface XX and free to be moved along that surface. The upper surface mo is inclined at an angle with the horizontal. S is a slide which may move up or down in the guides G, the lower end being inclined or beveled at the same angle as the upper surface of P, on which it rests.

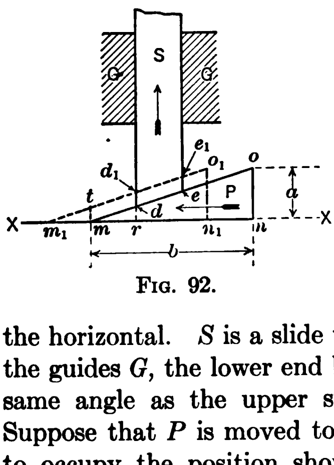


FIG. 92.

Suppose that P is moved to the left a distance mm_1, so as to occupy the position shown by the dotted lines. It is evident that S is forced up a distance dd_1. If the length b and height a of P are known, it is possible to calculate the amount S will move for any known movement of P. Draw a vertical line mt meeting m_1o_1 at t. Then $mt=dd_1$ since

they are sides of a parallelogram. The triangles m_1mt and $m_1n_1o_1$ are evidently similar.

Therefore,

$$\frac{mt}{o_1n_1}=\frac{mm_1}{m_1n_1}.$$

But

$$o_1n_1=on,$$

$$mt=dd_1$$

and

$$m_1n_1=mn.$$

Therefore

$$\frac{dd_1}{on}=\frac{mm_1}{mn},$$

or

$$dd_1=mm_1\times\frac{on}{mn},$$

or, in words, *the distance the slider rises is equal to the distance the wedge moves multiplied by the ratio of the height of the wedge to its length.* . . . . (43)

It should be noticed that this rule holds true *only when* the slider S moves in a direction perpendicular to the surface

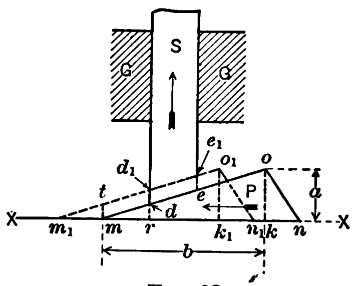


FIG. 93.

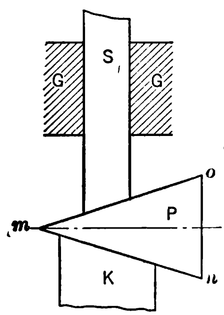


FIG. 94.

along which the wedge slides and the angle mno is a right angle.

In Fig. 93 a wedge is shown in which the end no is not perpendicular to mn. The same method of calculating

the rise of the slider S would be used as in the previous case except that the vertical height *ok* is used in place of the length *no*.

The wedge in Fig. 94 is itself raised when pushed to the left, due to its sliding upon the inclined stationary surface of K, and carries S up with it. It also gives an additional rise to S due to the slant of the surface *mo*. The resultant rise of S is, therefore, the sum of the two.

92. Screw Threads. If the top surface *mo* of the wedge shown in Fig. 92 is assumed to be covered with a very thin strip of flexible material and this strip is wound around a cylinder whose circumference is equal to the length b, the angle of inclination with the horizontal remaining the same, it will assume a helical form as shown in Fig. 95. If the

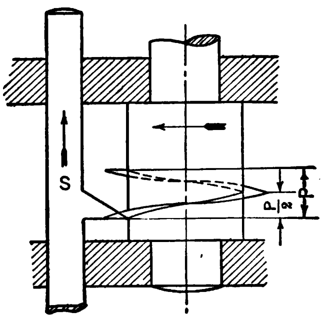


FIG. 95.

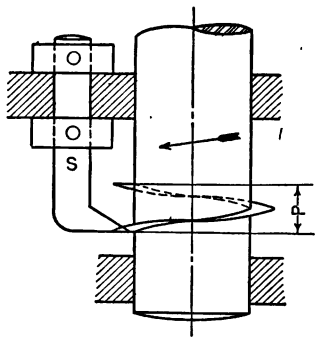


FIG. 96.

slide S has a point which reaches out and rests on the top surface of the strip, and the cylinder is turned in the direction of the arrow, S will be raised. The action of the helical surface on S is exactly similar to the action of the wedge in Fig. 92. One complete turn of the cylinder will raise S a distance of P. One-half a turn will raise S a distance $\frac{P}{2}$, and so on. In Fig. 96 a similar arrangement is shown, except that, in this case, S is stationary and the cylinder is free to

move endwise as well as turn, the weight of the cylinder resting on the point of S through the helical blade. Now, if the cylinder is given one turn in the same direction as before, it will be lowered a distance P.

Fig. 97 shows a cylinder with a strip wound around it in the same way as in the preceding figures only the strip

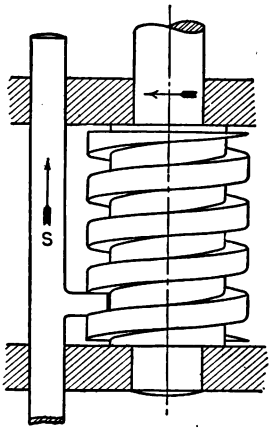


FIG. 97.

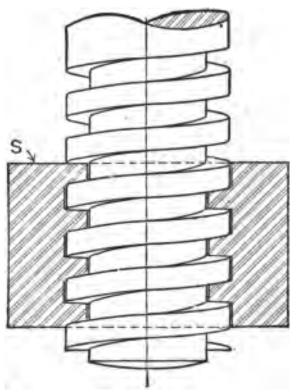


FIG. 98.

here is very much thicker and is wound around several times. In actually making such a cylinder of metal a solid cylinder of diameter D would be taken and a helical groove cut around it. The metal left between the successive turns of the groove thus constituting the "helical strip." A cylinder so formed is called a **screw,** the projecting part which we have called the helical strip, being known as the **screw thread.** The action of such a thread on its follower is exactly the same as just described for Fig. 95 or 96. Instead of using a single projecting point or surface for the thread to act against, as has been assumed in these figures, a hole with a corresponding thread inside it is formed, this thread being of the proper size, slant, and shape to just fit into the grooves

of the screw. The piece which contains such a hole is known as a **nut**. (See Fig. 98.)

93. Forms of Screw Threads. There are several different forms of threads in general use. The more common forms are shown in Figs. 99 to 102. In Fig. 99 is shown the **square thread** used for supporting or moving a load as in a jack-screw. In Fig. 100 is shown the thread ordinarily

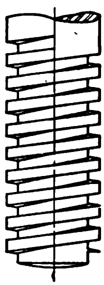


FIG. 99.

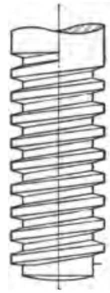


FIG. 100.

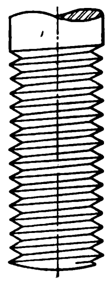


FIG. 101.

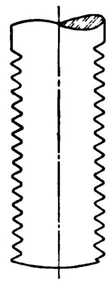
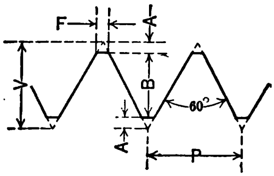


FIG. 102.

known as the **Acme thread** which is similar to the square thread except that its sides slope slightly, giving a stronger thread and making it possible to open and close a split nut around it. Such a thread is used on the lead screw of a lathe and in similar places where the screw moves the carriage and where it is necessary to separate the two halves of the nut on which it acts when it is desired to break the connection between the screw and the carriage.

Figs. 101 and 102 show the **V thread**, which is the kind commonly used on bolts, machine screws, and, in fact, for most purposes where the screw and nut serve for holding purposes. It is also used in light apparatus for causing motion. These two forms are alike except for a slight difference in the angle of the sides and a difference in shape at the point and root. Figs. 103 to 106 with the accompanying tables show the shapes and standard proportions of the above forms of threads.

TABLE V.—U. S. STANDARD SCREW THREADS



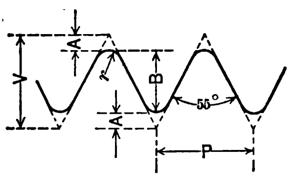
$$A=\frac{V}{8} \qquad F=\frac{P}{8},$$

$$B=\tfrac{3}{4}V=\tfrac{5}{8}P \text{ nearly.}$$

FIG. 103

Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.
$\frac{1}{4}$	20	$\frac{3}{4}$	10	$1\frac{1}{2}$	6	3	$3\frac{1}{2}$	5	$2\frac{1}{2}$
$\frac{5}{16}$	18	$\frac{13}{16}$	10	$1\frac{5}{8}$	$5\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{2}$	$5\frac{1}{4}$	$2\frac{1}{2}$
$\frac{3}{8}$	16	$\frac{7}{8}$	9	$1\frac{3}{4}$	5	$3\frac{1}{2}$	$3\frac{1}{4}$	$5\frac{1}{2}$	$2\frac{3}{8}$
$\frac{7}{16}$	14	$\frac{15}{16}$	9	$1\frac{7}{8}$	5	$3\frac{3}{4}$	3	$5\frac{3}{4}$	$2\frac{3}{8}$
$\frac{1}{2}$	13	1	8	2	$4\frac{1}{2}$	4	3	6	$2\frac{1}{4}$
$\frac{9}{16}$	12	$1\frac{1}{8}$	7	$2\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{1}{4}$	$2\frac{7}{8}$		
$\frac{5}{8}$	11	$1\frac{1}{4}$	7	$2\frac{1}{2}$	4	$4\frac{1}{2}$	$2\frac{3}{4}$		
$\frac{11}{16}$	11	$1\frac{7}{8}$	6	$2\frac{3}{4}$	4	$4\frac{3}{4}$	$2\frac{5}{8}$		

TABLE VI.—WHITWORTH OR ENGLISH STANDARD SCREW THREAD



$$A=\frac{V}{6}=.16P,$$

$$B=.64P \qquad r=.137P.$$

FIG. 104.

Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.	Dia. Screw.	Threads per In.
$\frac{1}{4}$	20	$\frac{5}{8}$	11	1	8	$1\frac{3}{4}$	5	3	$3\frac{1}{2}$
$\frac{5}{16}$	18	$\frac{11}{16}$	11	$1\frac{1}{8}$	7	$1\frac{7}{8}$	$4\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{1}{4}$
$\frac{3}{8}$	16	$\frac{3}{4}$	10	$1\frac{1}{4}$	7	2	$4\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{7}{16}$	14	$\frac{13}{16}$	10	$1\frac{7}{8}$	6	$2\frac{1}{4}$	4	$3\frac{3}{4}$	3
$\frac{1}{2}$	12	$\frac{7}{8}$	9	$1\frac{1}{2}$	6	$2\frac{1}{2}$	4	4	3
$\frac{9}{16}$	12	$\frac{15}{16}$	9	$1\frac{5}{8}$	5	$2\frac{3}{4}$	$3\frac{1}{2}$		

TABLE VII.—ACME THREADS

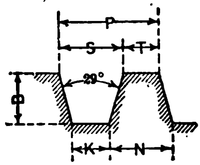


FIG. 105.

Threads.	B	T	K	S	N	Threads.	B	T	K	S	N
16	.048	.023	.018	.039	.044	$1\frac{3}{5}$	.323	.232	.226	.393	.399
10	.060	.037	.032	.063	.068	$1\frac{1}{2}$	.343	.247	.242	.419	.425
9	.066	.041	.036	.070	.075	$1\frac{5}{11}$	.354	.255	.250	.433	.438
8	.073	.046	.041	.079	.084	$1\frac{1}{3}$	.385	.278	.273	.472	.477
7	.081	.053	.048	.090	.095	$1\frac{3}{13}$	.416	.301	.296	.511	.516
6	.093	.062	.057	.105	.110	$1\frac{1}{7}$	.448	.324	.319	.551	.556
$5\frac{1}{2}$	.104	.070	.064	.118	.123	$1\frac{1}{15}$	.479	.348	.342	.590	.595
5	.110	.074	.070	.126	.131	1	.510	.371	.366	.629	.635
$4\frac{1}{2}$	.121	.082	.077	.140	.145	$\frac{16}{17}$	.541	.394	.389	.669	.674
4	.135	.093	.088	.157	.163	$\frac{8}{9}$	.573	.417	.412	.708	.713
$3\frac{1}{2}$	.153	.106	.101	.180	.185	$\frac{16}{19}$	.604	.440	.435	.747	.753
$3\frac{1}{5}$	.166	.116	.111	.197	.202	$\frac{4}{5}$	.635	.463	.458	.787	.792
3	.177	.124	.118	.210	.215	$\frac{16}{21}$	.666	.487	.481	.826	.831
$2\frac{2}{3}$	.198	.139	.134	.236	.241	$\frac{8}{11}$	.698	.510	.505	.865	.870
$2\frac{1}{2}$	.210	.148	.143	.252	.257	$\frac{16}{23}$	.729	.533	.528	.905	.910
$2\frac{2}{7}$	.229	.162	.157	.275	.280	$\frac{2}{3}$	.760	.556	.551	.944	.949
2	.260	.185	.180	.315	.320	$\frac{8}{13}$	.823	.603	.597	1.023	1.028
$1\frac{7}{9}$	.291	.209	.203	.354	.359	$\frac{4}{7}$	.885	.649	.644	1.101	1.106
.....	..	..	..	..	..	$\frac{8}{15}$	.948	.695	.690	1.180	1.185
.....	..	..	..	..	..	$\frac{1}{2}$	1.010	.741	.736	1.259	1.264

SQUARE THREADS

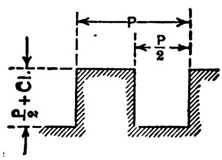


FIG. 106.

Threads per inch may be about $\frac{3}{4}$ of the U. S. Standard on both Acme and Square.

94. Single and Multiple Threads. All of the threads shown in the preceding illustrations are single threads; that is, the threads are formed by the metal left between the successive turns of a single helical groove cut around and around the cylinder. If two parallel helical grooves are cut, the metal remaining will constitute a *double* thread; three parallel grooves will leave a *triple* thread, and so on. The single, double, and triple threads are illustrated in Figs. 107, 108, and 109, respectively. It will be noticed on the

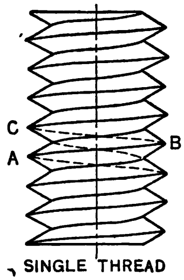


FIG. 107.

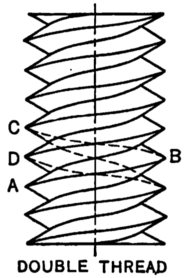


FIG. 108.

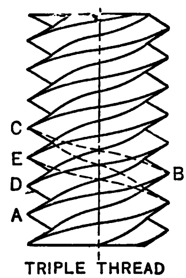


FIG. 109.

single thread (Fig. 107) that if the finger be placed on any point of the thread, as at A, and is moved along the thread until it has gone once around the screw, it will come to the point C. That is, in moving once around the screw the finger has advanced along the screw a distance AC. On the double thread (Fig. 108) if the finger starts at A and follows the thread once around, it will come to C, but this time there is a point D which lies between A and C. D is the point of the second or parallel thread. Similarly, if the finger follows a thread in Fig. 109 once around from A to C, two points D and E will lie between A and C. A multiple thread may be used when there is need for a fine thread having a large "lead" (see §95).

95. Pitch or Lead of a Screw. The distance AC which the thread advances along the screw in one turn around is

sometimes called the *pitch*. A better name, however, is the *lead*. This definition of lead applies equally to single and multiple threads, while the term *pitch* is sometimes applied to the distance from one point to the next, regardless of the condition of the screw being single or multiple. Lead is never used in this sense. If a nut is stationary and the screw is turned once around, it will move along through the nut a distance equal to the lead. If the screw is held from moving endwise but can turn, while the nut is held from turning but is free to move along the screw, one turn of the screw will move the nut a distance equal to the lead.

96. Threads per Inch. The size of a thread on a screw is commonly specified by stating the number of threads which the screw has in an inch of its length. For example, Fig. 110 represents the side of a screw with a scale laid

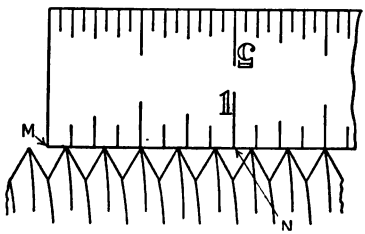


FIG. 110.

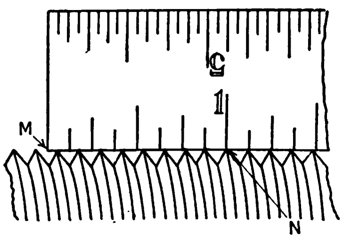


FIG. 111.

against it so that the line M is over the center of a groove. The line N, which is an inch from M, comes over the center of another groove or another turn of the same groove and there are five whole thread points between M and N. This screw would be described as a screw having five threads to the inch, no account being taken of its being single or double. In Fig. 111 the line M is placed over the center of a space and the line N happens to come over the center of a thread point with seven whole thread points between. There are, therefore, seven and one-half threads per inch on this screw.

97. Right-hand and Left-hand Threads. The thread may wind around the screw in such a way that it slants downward from right to left as one looks at it, as shown in Fig. 112, in which case it is called a *right-hand* thread; or, it may slant downward from left to right as one looks at it, as shown in Fig. 113, when it is called a *left-hand* thread. If the screw

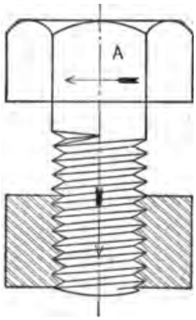


FIG. 112.

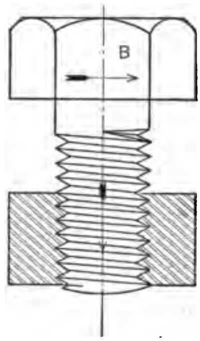


FIG. 113.

with the right-hand is turned in the direction of the arrow A, Fig. 112, it will move downward through the stationary nut, or if the screw cannot move endwise the nut will be drawn up. The screw with the left-hand thread would have to be turned in the direction of the arrow B (Fig. 113), to move downward or to draw the nut up. If one were looking at the end of a right-hand screw and turned it *right-handed* or *clockwise,* it would move away from him, whereas a left-handed screw looked at endwise and turned *left-handed* or *anti-clockwise* would move away.

98. Relation between the Speed of a Screw or Nut and the Speed of a Point on the Wrench or Handle. In Fig. 114 suppose the screw S is supported in a bearing. Collars H and B prevent it from moving endwise. The lead of the screw is P inches. S fits into a nut N which is free to slide along the guides G but is held from turning by the guides.

A crank with a handle K is fast to the end of the screw, the center of K being at a distance of R inches from the axis of the screw. It is now required to find a method of determining

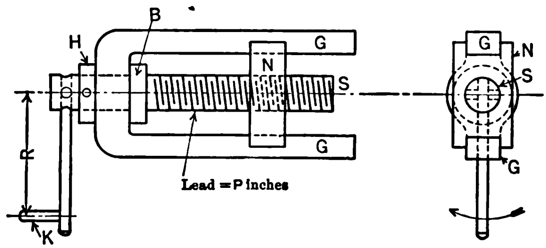


FIG. 114.

the relation between the linear speed of the handle K and of the nut N. If the crank is given one complete turn it will, of course, turn the screw once and the nut will move along the guides a distance P inches. While the crank turns once the center of K moves over the circumference of a circle whose radius is R, therefore it moves over a distance $2\pi R$ inches. Therefore the

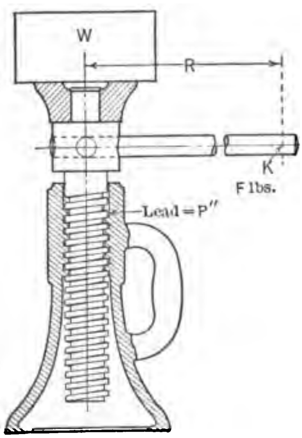


FIG. 115.

$$\frac{\text{Linear speed of } N}{\text{Linear speed of } K}=\frac{P}{2\pi R}. \quad (44)$$

Also, since the forces at the two points are inversely as the speeds, neglecting friction,

$$\frac{\text{Force at } N}{\text{Force at } K}=\frac{2\pi R}{P}. \quad (45)$$

In Fig. 115, which shows an ordinary jack-screw, the exact value of the speed ratio differs slightly from that expressed by Eq. (44). Here the point K at which the

force is applied rises with the screw so that in making a complete turn the point K moves over a helix whose diameter is $2R$ and whose pitch (or lead) is equal to that of the screw. The formula for the length of a helix is $\sqrt{\overline{2\pi R}^2+P^2}$ so that the actual speed ratio is

$$\frac{\text{Linear speed of } W}{\text{Linear speed of } K}=\frac{P}{\sqrt{\overline{2\pi R}^2+P^2}}. \quad . \quad . \quad (46)$$

The lead (P) is so small relative to R that the value

$$\sqrt{\overline{2\pi R}^2+P^2}$$

differs only very slightly from $2\pi R$. Accordingly, although Eq. (46) is the correct one, Eq. (44) is usually accurate enough for all practical purposes.

99. Differential Screws. Fig. 116 illustrates the style of screw known as a *differential screw.* A part S of the

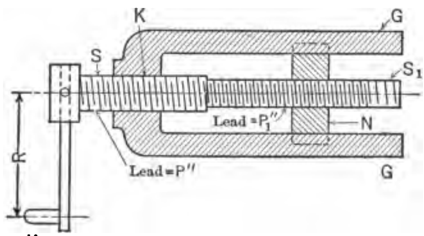


FIG. 116.

screw itself has a thread whose lead is P inches and fits into a nut T which is a part of the stationary frame. The other end S_1 of the screw has a different thread, of lead P_1, inches which fits the nut N. This nut may slide along the guides G but is held by the guides from turning. As the screw is turned the motion of the nut is the resultant of the movement of the screw S through the nut T and of the nut N along S_1. Suppose, for example, that $P=\frac{1}{2}$ in. and $P_1=\frac{7}{16}$ in., both being right-handed screws. If now the handle K

is turned right-handed, as seen from the left, the whole screw moves along through T toward the right $\frac{1}{2}$ in. and if it were not for the thread S_1 N would move to the right $\frac{1}{2}$ in. At the same time, however, S_1 has drawn N back upon itself $\frac{7}{16}$ in. so that the net movement of N toward the right is $\frac{1}{2}$ in. $-\frac{7}{16}$ in. or $\frac{1}{16}$ in. Again, suppose $P=\frac{1}{2}$ in. right hand and $P_1=\frac{7}{16}$ in. left hand. One turn of the handle in the same direction as before will advance S through T $\frac{1}{2}$ in. and at the same time carry N off S_1 $\frac{7}{16}$ in., so that the net movement of N to the right is $\frac{1}{2}+\frac{7}{16}$ in. or $\frac{15}{16}$ in. A device of this sort may be used for obtaining a very small movement of the nut for one turn of the screw without the necessity of using a very fine-threaded screw.

100. Examples on Velocity and Power of Screws.

Example 23. In Fig. 117, suppose it is required to find the load W, which, suspended from the nut N, can be raised by a force of 60 lb. applied at F. The screw has a lead of $\frac{1}{2}$ in. Assume that the friction loss is 40 per cent. Let $R=20$ in.

Solution. While the screw makes one turn F moves over a distance $2\pi 20=125.66$ in. and N rises $\frac{1}{2}$ in.

It has already been learned that the force multiplied by the distance over which it acts is equal to the weight lifted multiplied by the distance it moves.

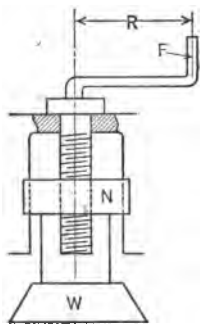


FIG. 117.

Therefore,

$$F\times 125.66 \text{ in.} = W\times \tfrac{1}{2} \text{ in.}$$

Since 40 per cent is lost in friction the net force is

$$.60\times 60=36 \text{ lb.}$$

Therefore,

$$36\times 125.66=W\times \tfrac{1}{2} \text{ in.},$$

or

$$W=9047.5 \text{ lb.}$$

The same result would be obtained by substituting directly in Eq. (45).

Example 24. In the jack-screw shown in Fig. 115, the lead of the screw is $\frac{1}{2}$ in. $R=3$ ft., 6 in. The force exerted at K is 100 lb. To find the weight W which could be lifted if friction were neglected.

Solution. Formula (46) applies in this case in finding the speed ratio, but formula (44) will be very nearly correct.

$$\frac{\text{Speed of } W}{\text{Speed of } K}=\frac{\frac{1}{2}\text{ in.}}{2\pi 42}=\frac{100}{W}.$$

Therefore,

$$W=2\pi 42\times 100\times 2=54779.$$

Example 25. In Fig. 118 $P_1=\frac{3}{16}$ in. right hand; $P_2=\frac{1}{8}$ in. right hand. To find how many turns of the hand wheel are required to lower the slide $\frac{1}{2}$ in., and to determine the direction the wheel must be turned.

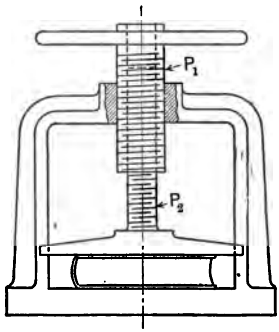


FIG. 118

Solution. Since the outer screw is right hand and has a lead of $\frac{3}{16}$ in. one turn of the wheel right-handed as seen from above will lower the outer screw $\frac{3}{16}$ in. At the same time, since the inner screw is also right-handed this one turn of the wheel will draw the inner screw into the outer one $\frac{1}{8}$ in. so that the resultant downward motion of the slide for one turn of the wheel is $\frac{3}{16}$ in. $-\frac{1}{8}$ in. $=\frac{1}{16}$ in. Therefore, to lower it $\frac{1}{2}$ in. the wheel must be turned right-handed as seen from above as many times as $\frac{1}{16}$ is contained into $\frac{1}{2}$ or 8 times.

101. Worm and Wheel. Fig. 119 is a picture of a worm and wheel mechanism mounted on a frame so as to be used as a model. The worm is merely a screw while the wheel is a gear with teeth so shaped that they mesh properly into the spaces of the worm thread. The worm may be right hand or left hand and single or multiple threaded.

Just as a screw when turned moves the nut along, so the worm when turned pushes the teeth of the worm wheel along, causing the wheel to turn. One turn of the worm will move a point on the pitch circle of the wheel over an arc equal in length to the pitch or lead of the worm. Therefore, in order to cause the wheel to make a complete turn the worm must turn as many times as the lead is contained into the circumference of the pitch circle of the wheel.

Let P represent the lead of the worm and D the pitch diameter of the wheel.

Then $$\frac{\text{Turns of worm}}{\text{Turns of wheel}}=\frac{\pi D}{P}, \qquad (47)$$

or

$$\text{Turns of wheel}=\text{Turns of worm}\times\frac{P}{\pi D}. \qquad (48)$$

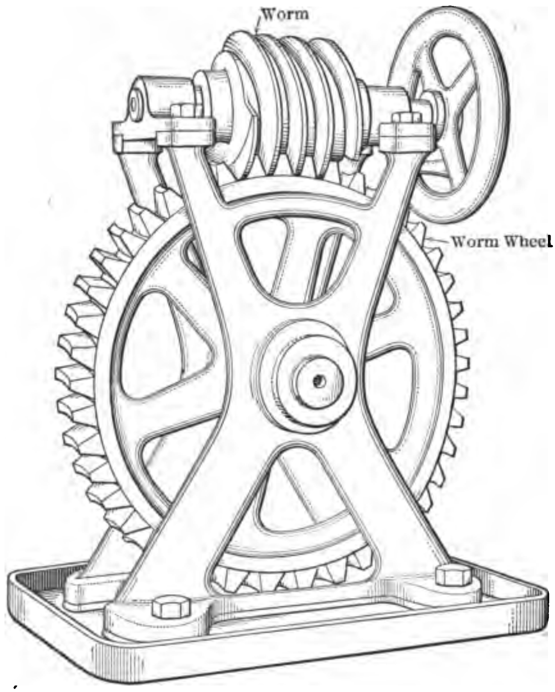


Fig. 119.

Again, the action of the worm on the wheel may be considered similar to the action of a rack on a gear. One turn of a single-threaded worm is the same as sliding a rack along a distance equal to the circular pitch of the wheel, or,

turning the wheel through $\frac{1}{T}$ part of a turn, where T represents the number of teeth in the wheel. One turn of a double-threaded worm corresponds to moving a rack along a distance equal to twice the circular pitch of the wheel, or turning the wheel through $\frac{2}{T}$ part of a turn.

Therefore,

$$\text{Turns of wheel} = \frac{1}{T} \times \text{turns of single-threaded worm}, \quad (49)$$

$$\text{Turns of wheel} = \frac{2}{T} \times \text{turns of double-threaded worm}, \quad (50)$$

$$\text{Turns of wheel} = \frac{3}{T} \times \text{turns of triple-threaded worm}, \quad (51)$$

Or, to express the same idea in more general terms,

$$\frac{\text{Turns of wheel}}{\text{Turns of worm}} = \frac{\text{Number of threads in worm}}{\text{Number of teeth in wheel}}. \quad (52)$$

It will be noticed that the angular speed ratio of a worm and worm wheel as expressed in Eq. (52) is the same as for spiral gears as expressed in Eq. (19). In fact the worm and wheel do not differ essentially from spiral gears, and it is not easy to determine where the line is drawn between spiral gears and worm and wheel. Perhaps a general method of distinguishing between the two is as follows:

In spiral gears each tooth on each gear is only a part of a helix of large lead, whereas in the worm and wheel the wheel teeth are short lengths of helices of very great lead while the worm itself is a gear of one, two or, at any rate, very few teeth, each of which winds around once or more times.

102. Examples on Worm and Wheel.

Example 26. In the worm and wheel mechanism shown in Fig. 120, let it be required to find the number of turns of the worm that would be necessary to turn 14 times.

Solution. Applying Eq. (47),

$$\frac{\text{Turns of worm}}{14}=\frac{\pi 10}{.628}.$$

Therefore $$\text{turns of worm}=\frac{14\pi 10}{.628}=700 \text{ approximately.}$$

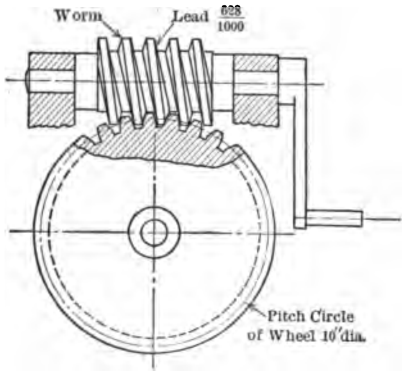


Fig. 120.

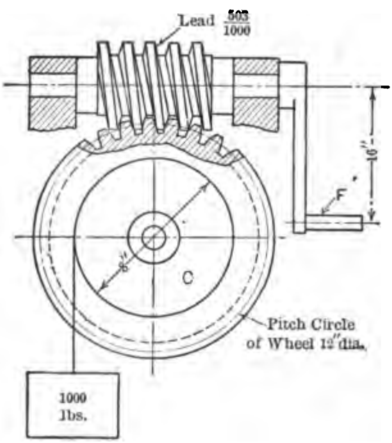


Fig. 121.

Example 27. The cylinder C, Fig. 121, is keyed to the same shaft as the worm wheel. It is required to find the force F which would be

necessary on the handle in order to raise the weight of 1000 lb. if friction is neglected.

Solution 1. First determine the ratio of the linear speeds of the weight and the point at which the force F is applied. If the worm is assumed to make one turn in a unit of time the handle will have a speed equal to the circumference of a circle whose radius is the distance from the axis of the worm to the center of the handle. Therefore, it would be $2\pi 16$ in. $=32\pi$ in. While the worm turns once the wheel will turn

$$\frac{\text{Lead}}{\text{circumference of wheel pitch circle}} \text{ or } \frac{.503}{\pi 12} \text{ turns.}$$

Since the cylinder C turns at the same angular speed as the worm wheel a point on its circumference will have a linear speed of $\frac{.503}{\pi 12}\times\pi 8$ $=.3353$ nearly.

Therefore, since the force is to the weight as the speed of the weight is to the speed of the point at which the force is applied,

$$\frac{F}{1000}=\frac{.3353}{32\pi}, \text{ or } F=3\tfrac{1}{3} \text{ lb.}$$

Solution 2. This problem might have been solved by a somewhat shorter method, as follows: Assuming the worm single-threaded, the wheel must have as many teeth as the lead is contained in the circumference of the pitch cirle, or $\frac{\pi 12}{.503}=75$ teeth. Therefore, one turn of the worm will cause the wheel and the cylinder C to make $\frac{1}{75}$ of a turn (see Eq. 49). Then, if the radius of the crank were equal to the radius of C the force F would be $\frac{1}{75}$ of the weight W. But since the radius of the crank is 4 times that of C the force F will be $\frac{1}{4}\times\frac{1}{75}$ of W or $\frac{1}{300}$.

Therefore $F=\frac{1000}{300}$ or $3\frac{1}{3}$ lbs.

The same result would be obtained if the worm were not single-threaded, provided the lead is the same. For example, assume the worm double-threaded, then the wheel would have $2\times\frac{\pi 12}{.503}$ or 150 teeth and one turn of the worm would cause the wheel to make $\frac{2}{150}$ of a turn (see Eq. 50), which equals $\frac{1}{75}$ as before.

CHAPTER VII

CAMS

103. A cam is a plate, cylinder, or other solid having a curved outline or a curved groove, which rotates about a

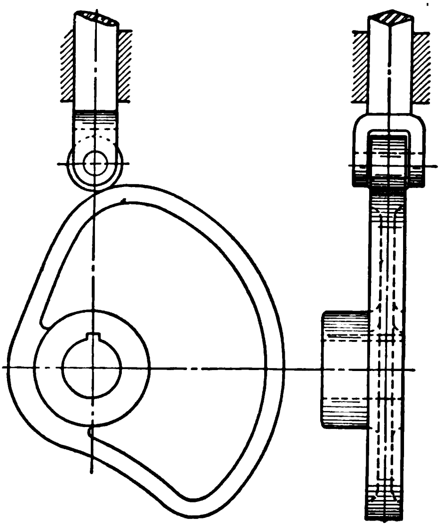


FIG. 122.

fixed axis and, by its rotation, imparts motion to a piece in contact with it, known as the follower.

Fig. 122 is a drawing of a cam known as a *plate cam*, and Fig. 123 a drawing of a cylinder containing an irregular groove and known as a *cylindrical cam*.

Very many machines, particularly automatic machines,

depend largely upon cams, properly designed and properly timed, to give motion to the various parts. A cam is

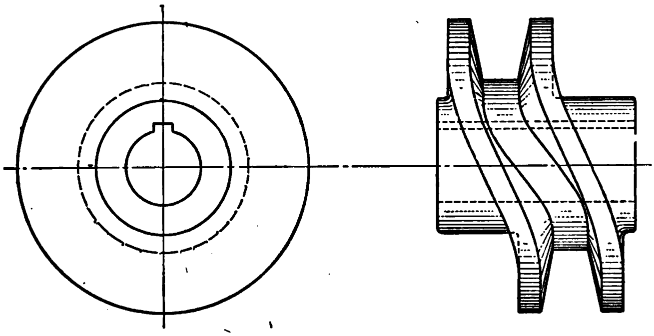


FIG. 123.

ordinarily designed for the special purpose for which it is to be used.

104. Plate Cams. A plate cam imparts motion to a follower guided so that it is constrained to move in a plane which is perpendicular to the axis about which the cam rotates; that is, in a plane coincident with or parallel to the plane in which the cam itself lies. The character of the motion given to the follower depends upon the shape of the cam. The follower may move continuously or intermittently; it may move with uniform speed or variable speed; or it may have uniform speed part of the time and variable speed part of the time. A knowledge of the various types of plate cams, and an idea of the manner of attacking the problem of designing a cam for any specific purpose, can best be obtained by studying a number of examples.

Example 28. A cam is to be keyed to the cam shaft (Fig. 124), which turns as indicated. The shape of the cam is to be such that the point of the slider S will be raised with uniform motion from A to B while the cam makes one-half a turn, and lowered again to the original

position during the second half turn of the cam. The cam shaft turns at uniform speed.

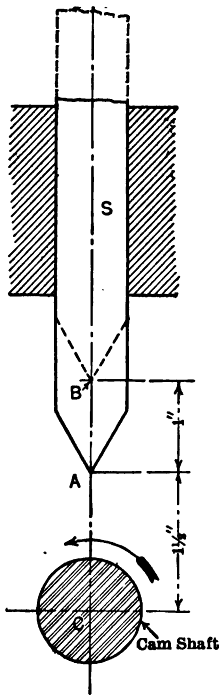


FIG. 124.

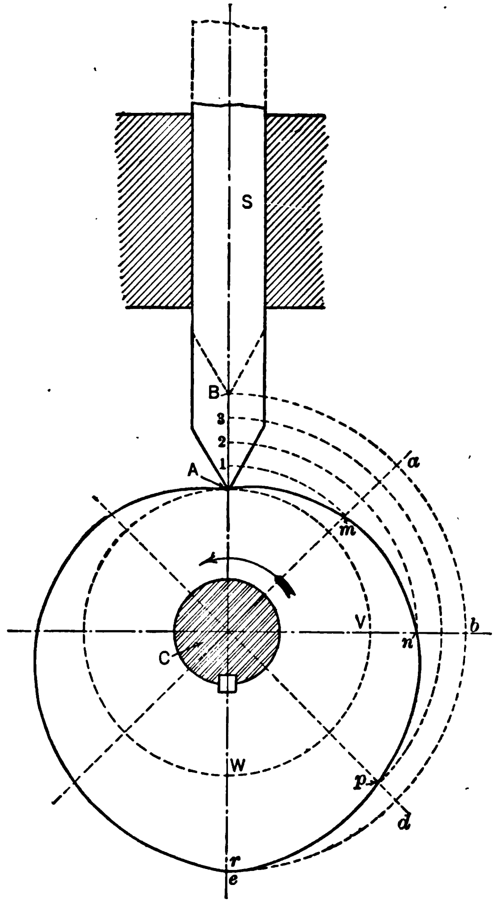


FIG. 125.

Solution. (Fig. 125.) Draw a circle through *A* with *C* as a center. Since the follower is to rise from *A* to *B* while the cam makes one-half a turn (or turns through 180°), and

since the cam shaft turns at uniform speed, divide one-half of the circle (AVW) into any number of *equal* angles by the lines Ca, Cb, Cd, and Ce. Four divisions are made in the illustration, although for accurate work a greater number would be desirable. The divisions are made on the side which is turning upward toward the follower, that is, back on the side *from* which the arrow is pointing. Now, divide the distance AB into as many parts as there are divisions in the angle AVW. Since the follower is to rise from A to B with uniform motion, the divisions of AB will be equal. That is, A to $1=1$ to $2=2$ to $3=3$ to B. When the cam has made one-fourth of a half revolution, the line Ca will be vertical. A point m on this line, found by swinging an arc through 1 with center C, will be the point on the cam which will be at the height $C1$ above the center when the cam has made one-fourth of one-half revolution. Similarly, n will be the point on the cam which will be at 2 when the cam has turned one-half of the half revolution. p and r are found in the same way, by drawing arcs through 3 and B cutting the lines Cd and Ce, respectively.

A smooth curve drawn through the points A, m, n, p, and r will be the correct outline for that portion of the cam which will raise the follower point from A to B as specified. Since the follower is to be lowered from B to A, also, with uniform motion during the remaining half turn of the cam, the other half of the cam outline will be a duplicate of that already found.

Example 29. Data the same as for Example 28, except that the follower, instead of having a point shaped as in that case, has a roller, as shown in Fig. 126, on which the cam acts. The construction is shown in Fig. 127. It is necessary first to find the outline of the cam for a follower like that in Fig. 125, the point of the follower being assumed to be at the center A of the roller, Fig. 127. The construction of this curve is exactly the same as explained for Fig. 125 and is lettered the same in Fig. 127, the curve itself being drawn

as a dot and dash line. This is called the *pitch line* of the cam. The next step is to set a compass to a radius equal to the radius of the roller, and with centers at frequent intervals on the pitch line, draw arcs as shown dotted. The true cam

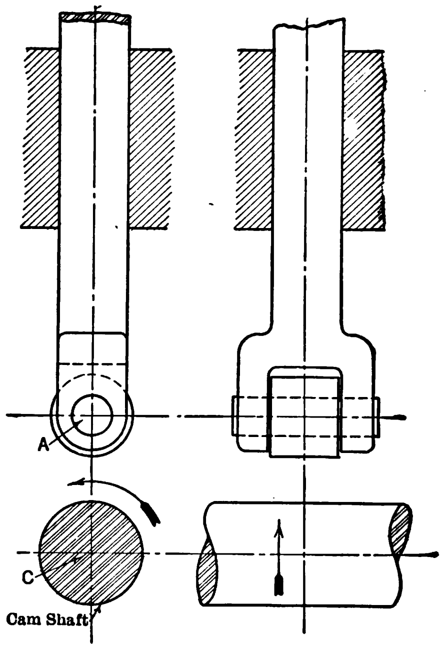


FIG. 126.

outline is a smooth curve drawn *tangent* to these arcs. It should be noted that the point of tangency will not necessarily lie on the line joining the center of the arc to the center of the cam. For instance, consider the arc drawn with n as a center. The cam curve happens to strike this arc at y, not at the point where the arc cuts the line Cb.

Example 30. Given a follower with a roller as shown in Fig. 128. The lowest position of the center oft he roller is a distance N above the center of the cam shaft, and the line

AB along which the center of the roller is guided is a distance *D* to the right of a vertical line through *C*. That is,

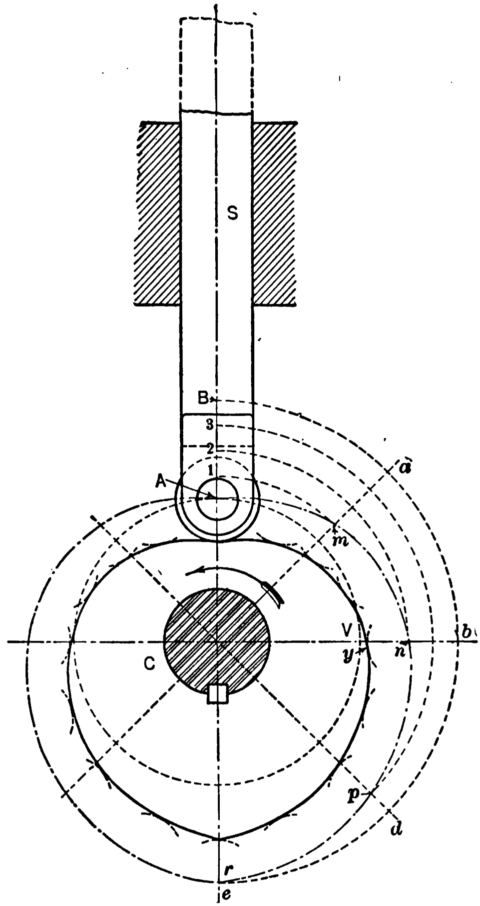


FIG. 127.

the center of the cam shaft is offset a distance *D* to the left of the line of motion of the center of the follower. To draw the outline of a plate cam which, by turning as shown by the

arrow, shall raise the center of the roller from A to B with uniform motion while the cam makes one half a turn, then lower it again to A during the second half revolution of the cam.

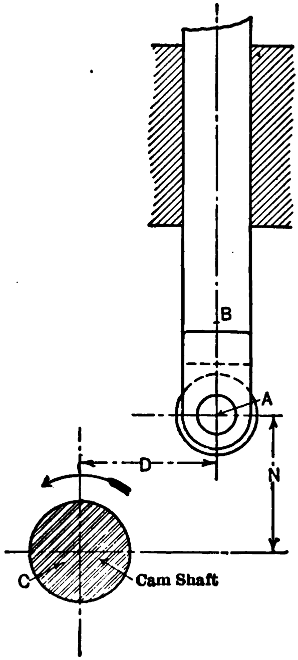


FIG. 128.

Solution. Fig. 129 shows the solution of this problem. Starting with C, locate the center A by measuring a distance D to the right of C and a distance N above C. Draw a line Ck through C and A. Since the upward motion is to take place during one-half turn of the cam, measure back 180° from Ck and draw Ce (that is $kACe$ is a straight line). Divide the angle kCe into any convenient number of equal parts as before (in this case four) by the lines Ca, Cb, Cd. Divide AB into the same number of equal parts, since the follower is to rise with uniform speed. From C as a center swing an arc through 1 cutting Ck at 5. Cut Ca with the same arc at 9. Make the length 9–10 equal to 5–1. Then 10 is one point on the pitch line of the cam. In the same way point 12 is found by making arc 11–12 equal to arc 6–2, and, similarly, all the way around. The true cam outline is found as before by drawing arcs with radius equal to the radii of the roller, and with centers on the pitch line, and then drawing a smooth curve tangent to these arcs.

Example 31. Fig. 130 shows the method of laying out a cam to move a follower from A to B with uniform motion during one-quarter turn of the cam, hold it at B during one-

quarter turn, lower it again to A during one-quarter turn, and allow it to remain at A during the last quarter turn; the cam to turn in the direction of the arrow. Starting with C and A as in the preceding figure, draw the line kAC. It is convenient, for the purpose of dividing up the angles, to draw a circle through A with C as a center. With CAk as a starting or reference line divide the circle into four equal

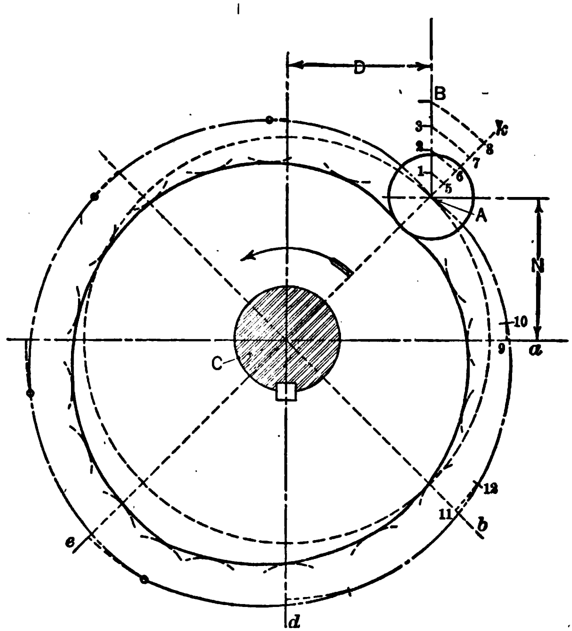


FIG. 129.

parts by the lines Ce, Cf and Cn. The angle kCe is the angle through which the cam turns while the follower is being raised. Divide this angle into equal parts, and find points on the cam pitch line as described in Example 30, the only difference here being that the angles kCa, kCb, etc., are smaller. Point 16 is the last point thus found. Since the follower is to remain at rest at B during the next quarter

turn of the cam, the pitch line of the part of the cam which holds it up there will be an arc of a circle drawn through 16 with C as a center. This part of the cam will extend from 16 to 18, the point 18 being found by extending the circle to

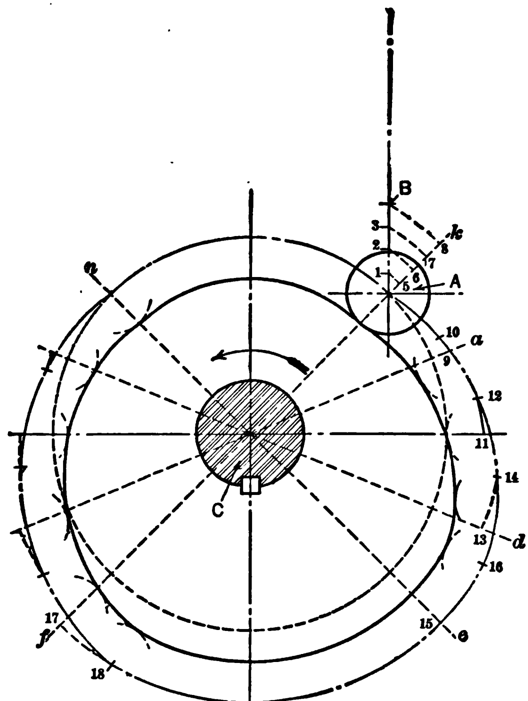


FIG. 130.

cut Cf at 17 and making the distance 17–18 equal to 15–16. The construction for completing the pitch line is exactly similar, and the cam curve proper is obtained as described in the previous examples.

Example 32. Fig. 131 is a cam which raises the center of the roller from A to B with harmonic motion during one-third of a turn, allows it to drop to its original position

instantly, and holds it there during the remaining two-thirds of a turn. The angle kCf through which the cam turns to raise the roller, is laid off (120°) and divided into an *even* number of *equal* parts. Since the roller is to rise with

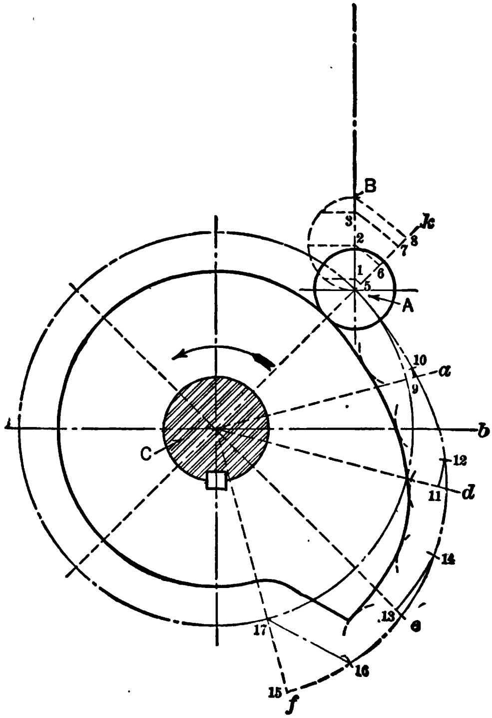


FIG. 131.

harmonic motion, a semicircle is drawn with AB as a diameter, and the circumference of this semicircle is divided into as many equal parts as there are divisions in the angle kCf. From the points of division on this semicircle perpendiculars are drawn to the line AB, meeting it at points 1, 2, 3. These points are the points of division of AB to be used

in finding the pitch line of the cam, which is found as previously described. The last point on the part which raises the follower is 16. Since the follower is to drop instantly, draw a straight line from 16 to 17, the point where an arc through A cuts Cf. The remainder of the pitch line is a circle about C through 17 around to A.

Example 33. In Fig. 132 a cam is to be placed on the shaft at C to act on a roller centered at A on the rocker ART.

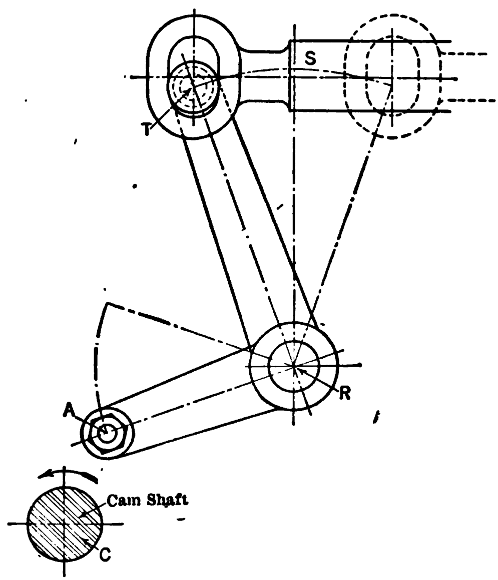


FIG. 132.

Another roller centered at T on the rocker fits a slot in the slider S. The cam is to be of such shape that, by turning as shown by the arrow, it will move the slider with harmonic motion to the position shown dotted during one-half turn of the cam in the direction indicated, and allow it to return to its original position, with harmonic motion, during the remaining half turn. Fig. 133 shows the construction.

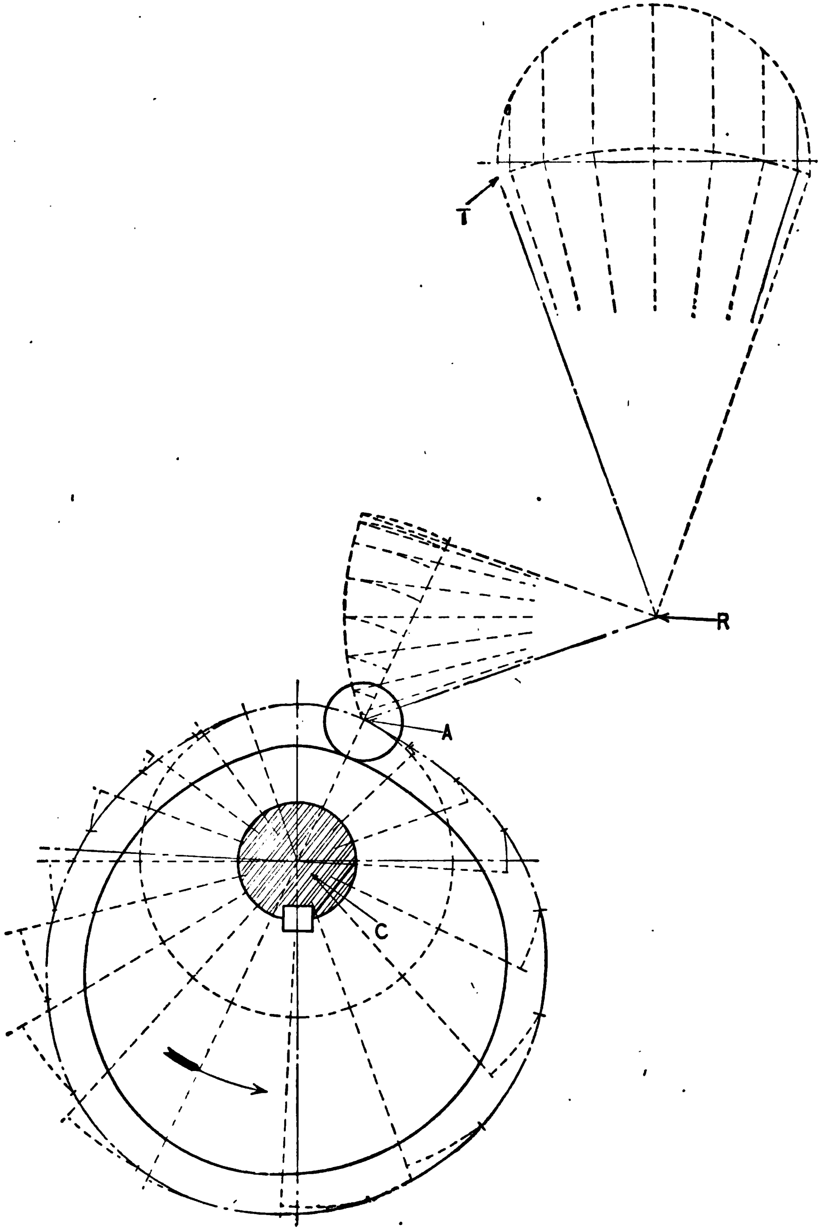


FIG. 133.

105. Positive Motion Plate Cams. It will be noticed that in each of the cams which have been discussed, the follower

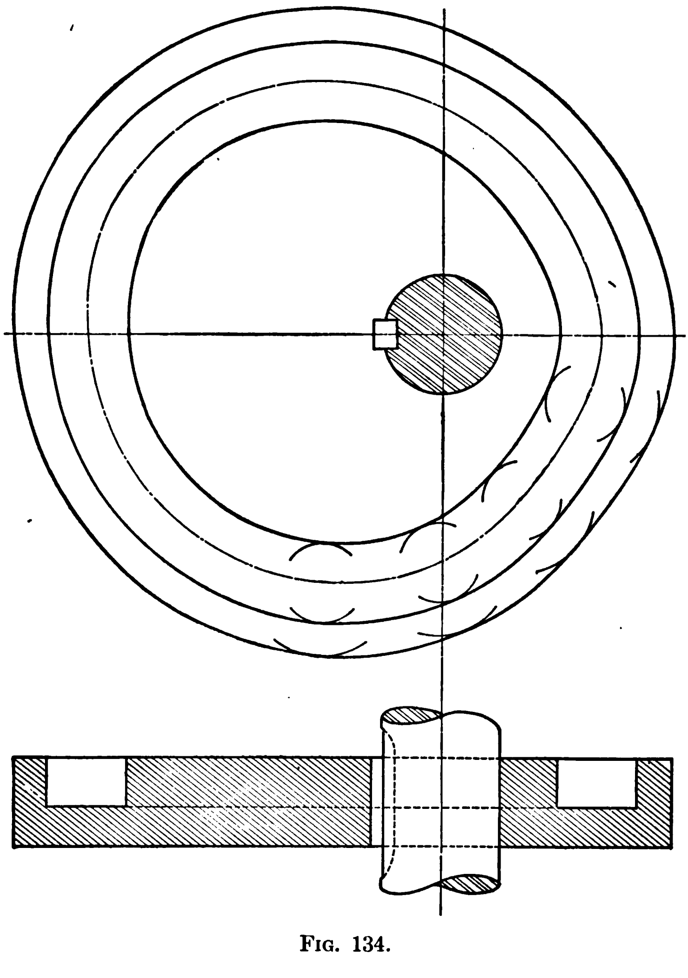


FIG. 134.

must be held in contact with the surface of the cam by some external force such as gravity, or a spring. The cam can

only force the follower away from the cam shaft, while some outside force must bring it back. In case it is desired to make the cam positive in its action in either direction without

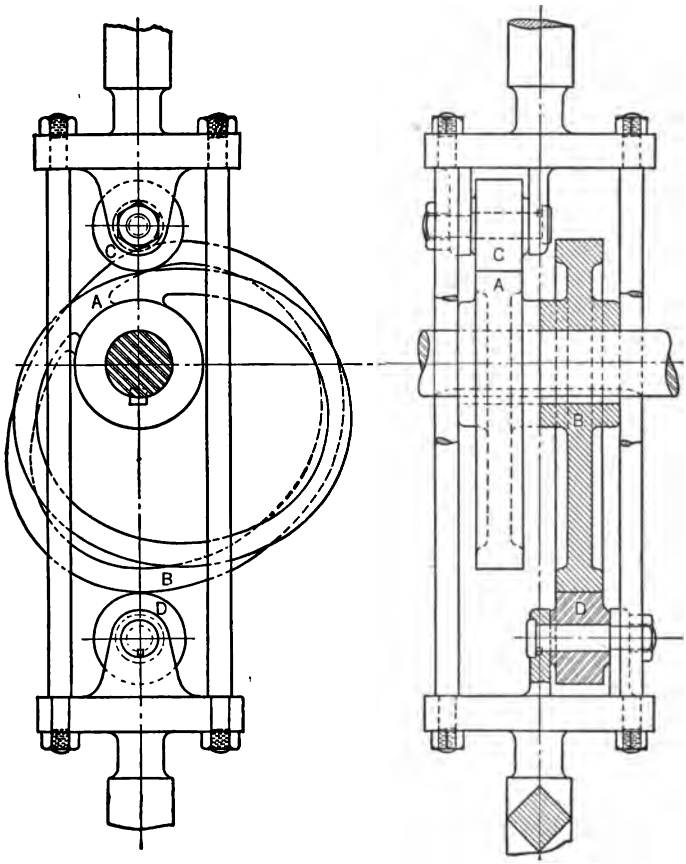


FIG. 135.

depending upon external force, the cam must be so constructed as to act on both sides of the follower's roller, or there must be two rollers, one on either side of the cam.

Fig. 134 shows a cam designed to give the same motion to the same follower as in Fig. 133. In Fig. 134, however, the pitch line of the cam is made the center line of a groove of a width equal to the roller diameter, enabling the cam to move the roller in either direction. Fig. 135 shows another style of positive motion cam. The follower consists of a framework carrying two rollers, one, roller C, resting on cam A, which is designed so as to give whatever motion is desired for the follower. The other, roller D, rests on cam B, which is designed to be in contact with roller D, the position of the latter depending in turn upon the position of the roller C. It would be possible to have both rollers touching the same cam, but in that case the movement of the follower could only be chosen for one-half a turn of the cam, the other half being determined by the shape of the cam necessary to be in contact with both rollers.

106. Plate Cam with Flat Follower.—Example 34. The follower for the cam shown in Fig. 136 has a flat plate at its end instead of a roller. The cam is so designed that, when it turns right-handed, the follower will be raised with harmonic motion while the cam makes one-third of a turn, then remains at rest during the next third of a turn of the cam and be lowered with harmonic motion during the remaining third of a turn.

If the center line of the guides in which the follower moves did not pass through the center of the cam, the shape of the cam would not be affected.

Solution. The method of construction is as follows: Assuming that the follower is shown in its lowest position, measure up along a vertical line passing through the center of the cam the distance $o8$ which the follower is to move. Divide this into any even number of harmonic divisions, eight being used in the drawing. Lay back the angle oEm equal to the angle through which the cam turns while the follower is being lifted. Divide oEm into as many equal angles as there are harmonic divisions in the line $o8$. Through

point 1 swing an arc with E as a center cutting the first radial line at w; through w draw a line perpendicular to Ew. Through 2 draw an arc cutting the second radial line at v and draw through v a line perpendicular to Ev. In a similar way draw perpendiculars to Eu, Et, Er, Ep, En, and Em.

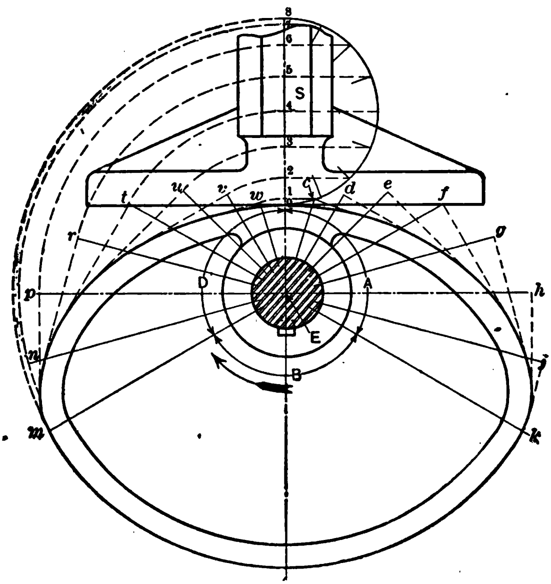


FIG 136.

A smooth curve tangent to all of these perpendiculars will be the outline of that portion of the cam which raises the follower.

Since the follower is to remain at rest while the cam turns through the next 120° the outline between the line Em and the line Ek 120° away from Em will be an arc of a circle through m with E as a center.

The outline of the portion of the cam which lowers the follower is found in a manner similar to that described for raising it.

107. Plate Cam with Flat Rocker.—Example 35. The cam in Fig. 137 actuates the follower *S* through the rocker *R* which is pivoted at *P*. *S* slides in guides, and is to remain still while the cam makes a quarter turn right-handed, then to rise to the upper dotted position with harmonic motion during a quarter turn of the cam. During the next quarter

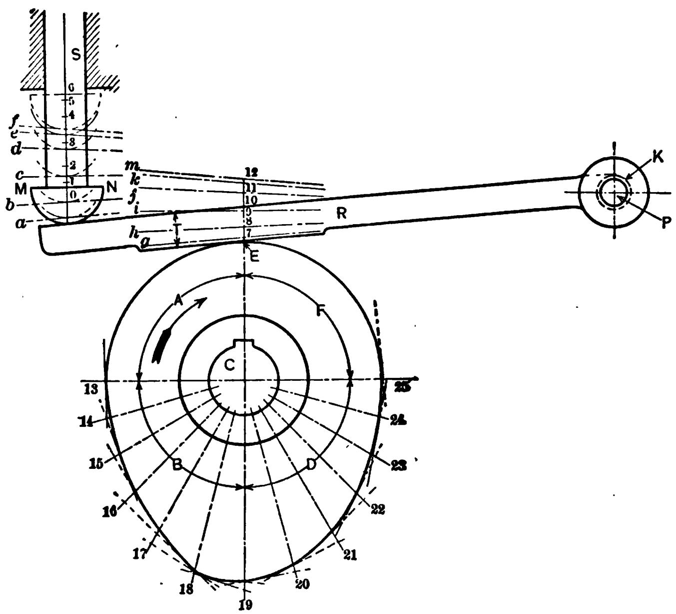


FIG. 137.

turn the follower is to drop with harmonic motion to its original position, and remain at rest during the last quarter turn. The foot of the follower is a semicircle with center at *o*, resting on the upper flat surface of the rocker. To find the cam outline, first divide the distance *o*6 into harmonic spaces, six being used in this case. These points of divisions are the successive positions of the center of the semicircle. Draw arcs of the circle with each of the points,

1, 2, 3, 4, 5, 6, as centers. Draw the dotted circle K tangent to the upper surface of the rocker produced. Next, draw the lines a, b, c, d, e, and f tangent to circle K and to the arcs drawn at 1, 2, 3, 4, 5, and 6 respectively. Parallel to, and at a distance T from lines a, b, c, etc., draw lines g, h, i, j, etc., cutting the vertical line through the cam center C at 7, 8, 9, 10, 11, and 12.

Since the follower is to remain at rest during a quarter turn of the cam, the outline of the cam over the angle A is an arc of a circle with radius CE.

Since the upward movement takes place during a quarter turn, or 90°, lay off angle B equal to 90° and divide it into as many equal angles as there are harmonic divisions in $o6$. Lay off $C14$ equal to $C7$ and through point 14 draw a line making the same angle with $C14$ that line g makes with CE. Draw similar lines through each of the other radial lines $C15$, $C16$, $C17$, $C18$, and $C19$. The cam outline will be a smooth curve tangent to all the lines which have been thus drawn.

A similar construction is used for finding the curve for the part of the cam which lowers the follower. The last part of the cam, over angle F, will be a circular arc to give the period of rest.

108. Cylindrical Cams.—Example 36. The general appearance of a cylindrical cam has already been shown (see Fig. 123). Fig. 138 gives dimensions for the hub and groove for a cylindrical cam which is to hold a follower still for $\frac{1}{8}$ turn of the cam, move it 1 in. to the right with gravity motion (see § 12) while the cam makes three-eighths turn, hold it still for one-eighth turn, and return it to its original position in three-eighths turn.

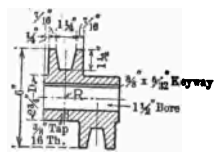


Fig. 138.

Solution. The solution of this problem is shown in Fig. 139. The upper left-hand view is an end view of the cam,

the upper right-hand view is a side elevation of the cam. To make the drawing, proceed as follows:

Locate the center line XX'. On the line XX' choose the point C at any convenient place and draw the circle K whose radius is equal to the outside radius of the cylinder. Also draw the dotted circle P with a radius equal to the outside radius minus the depth of the groove. Draw the vertical center line YY'. Lay back the angle YCB equal to $\frac{1}{8}$ of 360°, that is, 45°. This is the angle through which the cam will turn before the follower starts to move. Since the movement of the follower is to take place during the next three-eighths of a turn, the cam will turn through the angle BCY' to give the motion to the follower. Since the follower is to remain at rest during the next one-eighth turn, the angle $Y'CT$ equal to 45° will next be drawn, and the remaining angle TCY will be the angle through which the cam will turn to move the follower back to its original position. Now, draw the center line MN at any convenient distance on the right of the figure already drawn, and locate the point E on this line at a distance from XX' equal to the outside radius of the cylinder. On a horizontal line drawn through E locate the points F and G, each at a distance from E equal to the radius of the roller on which the cam is to act. Draw HJ parallel to FG at a distance from it equal to the depth of the groove. Through F and G draw lines to the point L where MN intersects the axis XX'. That portion of the line HJ intersected between FL and GL will be the width of the groove at the bottom. Before it is possible to proceed further in the construction of this side elevation of the cam, it is necessary to make a development of its outer surface. Draw the line $M'N'$ equal in length to the circumference of the cylinder.

Lay off $M'B'$ equal to the length of the arc YB and $B'Y'_2$ equal to the length of the arc BY'. Divide $B'Y'_2$ into any even number of equal parts, in this case eight, and letter points of division a', b', c', d', e', f', and g'. Through

the points thus found draw vertical lines. On the vertical line through M' lay off $M'8$ equal to the distance through

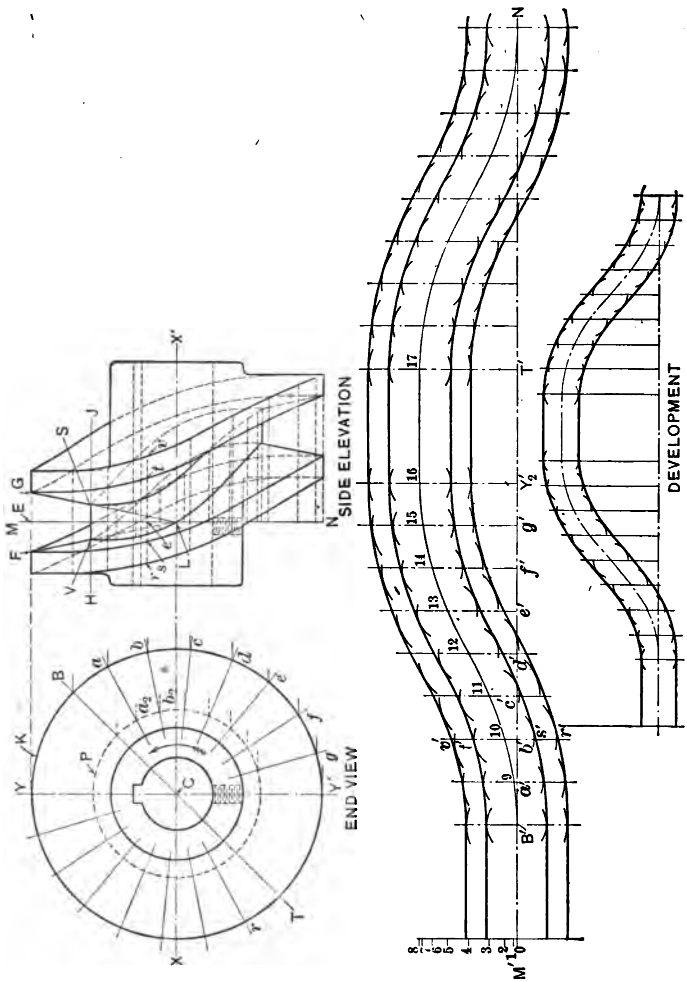


FIG. 139.

which the follower is to move, and divide $M'8$ into "gravity" divisions (see § 12), using as many divisions as there are

equal divisions in $B'Y'_2$. Mark the points thus found 1, 2, 3, 4, 5, 6, 7. From 1 project across to the vertical through a'. From 2 project to the vertical through b', and so on, thus getting the points 9, 10, 11, 12, 13, 14, 15, and 16. A smooth curve drawn through these points will be the development of the center line of that portion of the cam groove which moves the follower to the right. Make Y'_2T' equal to the length of the arc $Y'T$. The development of the center line of the groove between the verticals at Y'_2 and T' is a horizontal straight line. Since the return motion of the follower is a duplicate of the forward motion, the curve $17N'$, being a duplicate of the curve $B'16$, will be the development of the center line of that portion of the cam groove which moves the follower back to its original position.

The above construction gives a development of the center line of the groove on the outer surface of the cylinder. The lines forming the development of the sides of the groove are smooth curves drawn tangent to arcs, swung about a series of centers along the line $M'B'$ 16 17 N' with radii equal to the radius of the large end of the roller as shown in the drawing. Similar curves drawn tangent to arcs swung about the same centers with a radius equal to the radius of the large end of the roller plus the thickness of the flange forming the sides of the groove, will be the development of the outer corners of these flanges.

The development of the corners of the bottom of the groove is constructed in the same way, except that the length of the development is less, because it is a development of a cylinder of smaller radius.

The projections (on the side elevation) of the curves which have just been developed are drawn by finding the projections corresponding to points r', s', t', v', where these curves cut the vertical line, it being borne in mind that the vertical lines on the development really represent the developed positions of elements of the cylinder, drawn through points a, b, c, etc., which are found by dividing the arcs

BY' and TY into divisions equal to the divisions in $B'Y'_2$ and $T'N'$. The construction for the points r', s', t', and v' only will be followed through as the construction for all other points will be exactly similar. Through b on the end view draw an element of the cylinder across the side elevation. From e, where this element intersects MN, lay off et equal to $b't'$, ev equal to $b'v'$, to the right of MN since t' and v' are above $M'N'$, and es equal to $b's'$ and er equal to $b'r'$, to the left since s' and r' are below $M'N'$. The points r, s, t, v, are the projections of points corresponding to r', s', t', v'. Projections of all other points where the curves intersect the verticals on the development, are found in exactly the same way, and smooth curves drawn through the points thus found will be the projections of the corners of the groove, and of the flange enclosing the groove. The projections of the corners of the bottom of the groove are obtained in the same way also, using, of course, elements through a_2, b_2, etc., instead of a and b.

109. Multiple-turn Cylindrical Cam. Fig. 140 shows a cylindrical cam which requires two revolutions to complete the full cycle of motion of its follower. The method of designing such a cam would be similar in principle to that described for the simple cam in Fig. 139. The follower in a case like this may require a special form in order to pass properly the places where the groove crosses on itself.

110. Combinations of Two or More Cams. In various automatic machines the movements of parts which have to be timed with respect to each other are often obtained by the use of two or more cams properly designed, and properly adjusted to give each piece its desired motion at the required time. Fig. 141 shows how a cylindrical cam and a plate cam might be arranged to work in combination with each other. In this case the cylindrical cam makes two revolutions for every one of the plate cam. The cylinder R is caused to swing back and forth by the lever S which, in turn, is operated by the plate cam.

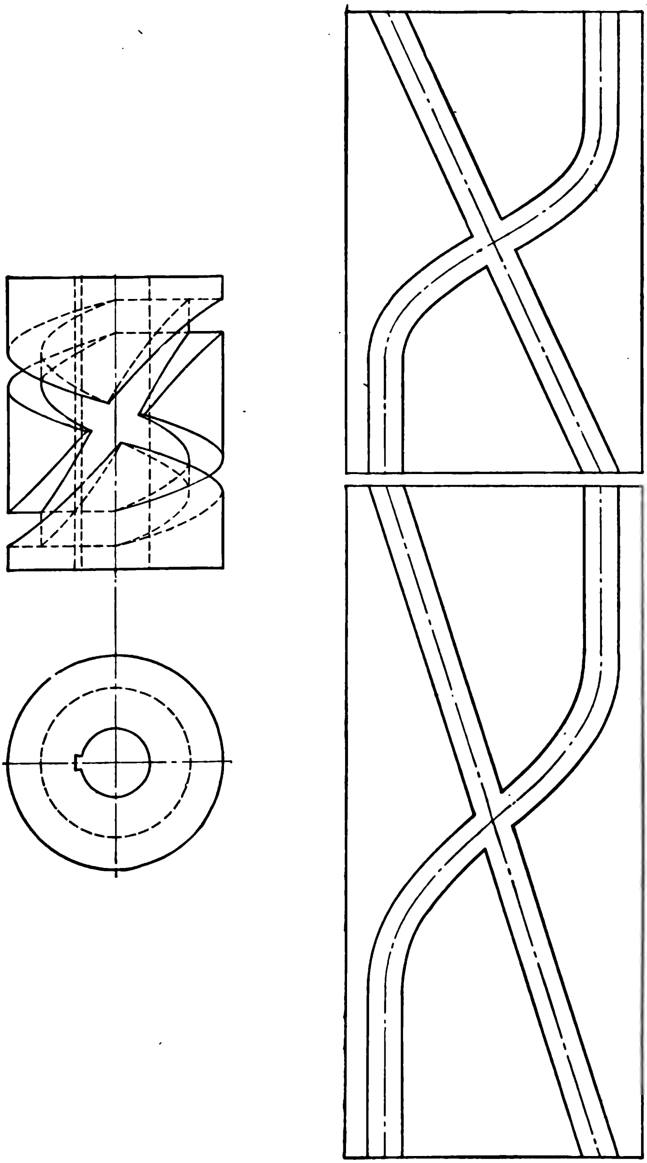


FIG. 140.

With the mechanism in the position shown, the cylindrical cam makes one-eighth turn in the direction shown, after which the pin T starts to move to the right with harmonic motion. T moves to the right the total distance of

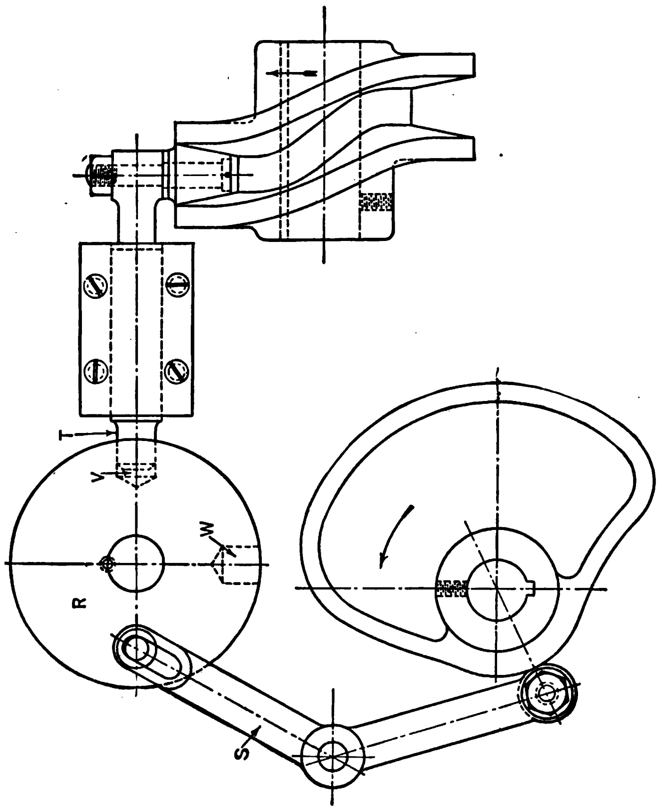


FIG. 141.

$1\frac{3}{8}$ in., during three-eighths of a turn of the cylindrical cam, after which it remains at rest for one-eighth turn of the cam, then returns to its original position during the remaining three-eighths turn. The plate cam is so designed that, turning left-handed as shown, the cylinder R begins to turn

after T has moved to the right $\frac{3}{4}$ of an inch. It continues to turn with "gravity" motion until T gets back again to within $\frac{3}{4}$ of an inch of its left-hand position.

The hole W will then be in the position now occupied by the hole V. R will then stop its motion and T will be inserted into the hole W. During the next revolution of the cylindrical cam T has a motion the same as before, and the plate cam swings the cylinder R back to its original position.

CHAPTER VIII

SIMPLE WHEEL TRAINS

111. Train of Wheels. A train of wheels is a series of rolling cylinders, cones, gears, pulleys or other similar devices serving to transmit power from one shaft to another.

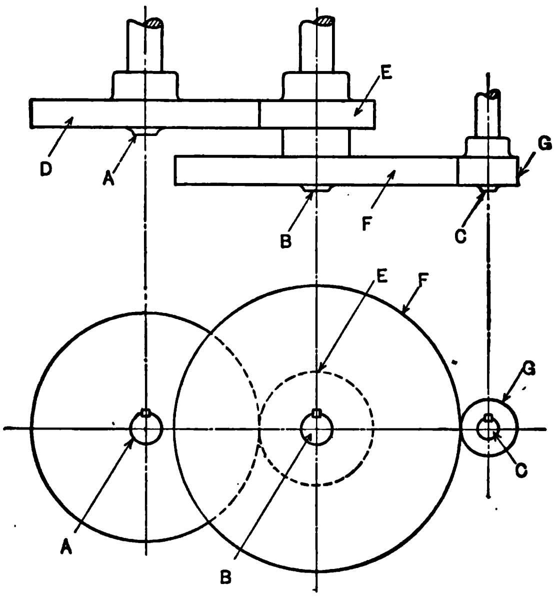


FIG. 142.

The examples of rolling cylinders, gears, etc., which have been discussed in earlier chapters are really wheel trains each involving only one pair of wheels. In Fig. 142

D is a gear fast to shaft *A*. *E* is a gear fast to shaft *B* and meshing with *D*. *F* is another gear also fast to shaft *B* and meshing with the gear *G* which is fast to shaft *C*. If

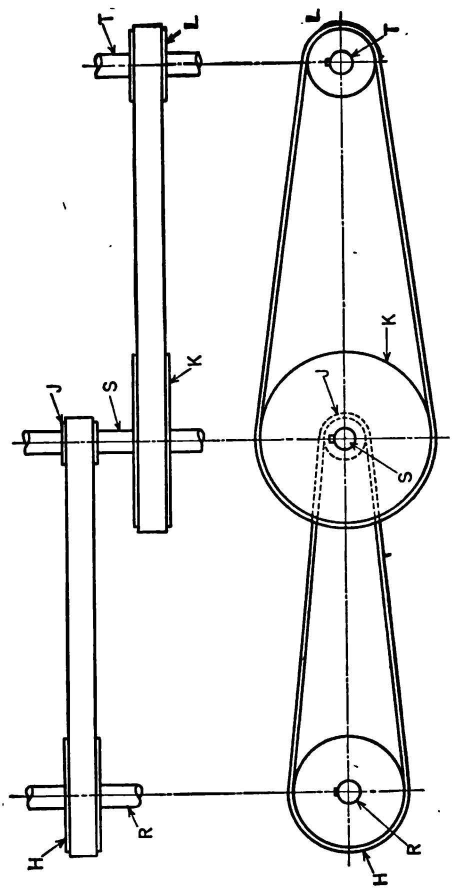


Fig. 143.

now the shaft *A* begins to turn, *D* will turn with it, and, therefore, cause *E* to turn. Since *E* is fast to the shaft *B* the latter will turn with *E*. Gear *F* will then turn at the

same angular speed as E and will cause G to turn, causing the shaft C to turn with it. That is, D drives E, and F, turning with E, drives G.

The above is an example of a simple train of gears consisting of two pairs. Fig. 143 shows an arrangement of pulleys similar in action to the gears shown in Fig. 142.

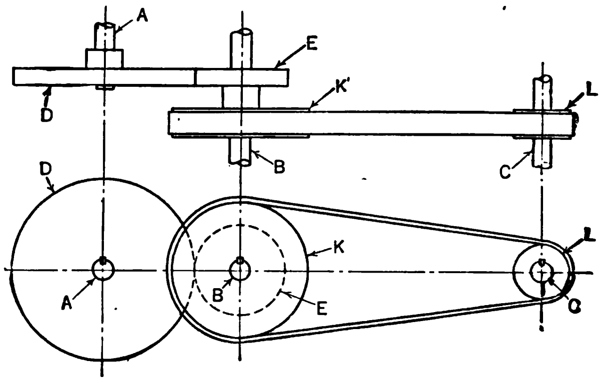


FIG. 144.

H is a pulley on the shaft R belted to the pulley J on shaft S. On the same shaft is another pulley K belted to the pulley L on shaft T.

Fig. 144 shows a train of wheels involving both gears and pulleys. In this case D is a gear on shaft A, meshing with and driving the gear E on shaft B. On the same shaft is pulley K, belted to the pulley L on shaft C.

112. Driving Wheel and Driven Wheel. Referring again to Fig. 142, the gear D by its rotation causes E to turn, therefore, D may be called the *driver* or *driving wheel*, and E the *driven wheel*. Similarly F, turning with E, is the driver for the wheel G. Hence, in any train such as here shown, consisting of three axes with two pairs of wheels, two of the wheels are drivers and two are driven wheels.

113. Idle Wheel. In Fig. 145, gear D drives E, which

in turn, drives F. E is, therefore, both a driven and a driving wheel. Such a wheel is called an *idle wheel.*

114. Train Value (Speed Ratio). The ratio of the angular speed of the last wheel of a train to the angular speed of the first wheel of the same train is called the *value of the train,* or *train value.* For example, if the shaft A in Fig. 142 makes 25 r.p.m. and the sizes of the several gears are such that shaft C makes 150 r.p.m, the value of the train would be $\frac{150}{25}=6$.

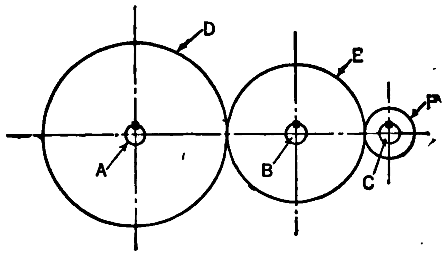


FIG. 145.

An inspection of the same figure will show that if A turns right-handed, B will turn left-handed and C will turn right-handed. The direction, then, of C is the same as that of A. The value of this train is then said to be positive, and will be indicated by putting a + sign in front of the number which indicates its value. If the number of wheels involved is such that the last shaft turns in the opposite direction from the first shaft, the value of the train will be said to be negative, which fact will be indicated by a − sign in front of the number indicating the train value.

115. Calculation of Speeds. Let it be assumed that the gears in Fig. 142 have teeth as follows:

D, 100 teeth	F, 125 teeth
E, 50 teeth	G, 25 teeth

It will also be assumed that shaft A makes 25 r.p.m. to find the speed of C. Since the speed of B is to the speed of A as

the teeth in D are to the teeth in E (see § 32), the revolutions of B will be equal to $25\times\frac{100}{50}$; also, since the speed of C is to the speed of E as the teeth in F are to the teeth in G, the speed of $C=25\times\frac{100}{50}\times\frac{125}{25}=250$. Expressing this as a formula,

The speed of the last shaft is equal to the speed of the first shaft

$$\times\frac{\text{Product of the teeth of all the drivers}}{\text{Product of the teeth of all the driven wheels}}. \qquad (53)$$

In the case of pulleys in Fig. 143 the principle is the same, except that diameters are used instead of numbers of teeth. Suppose that pulley H is 24 in. in diameter, J 8 in. in diameter, K 36 in. in diameter, and L 12 in. in diameter, then the speed of T will be equal to the speed of $R\times\frac{24\times36}{8\times12}$; that is, in the case of a train of pulleys:

The speed of the last shaft is equal to the speed of the first shaft

$$\times\frac{\text{the product of the diameter of all the driving pulleys}}{\text{the product of the diameter of all the driven pulleys}}. \qquad (54)$$

In a train consisting of a combination of gears and pulleys, as in Fig. 144

The speed of the last shaft is equal to the speed of the first shaft

$$\times\frac{\text{the product of the diameters and numbers of teeth of all the driving wheels}}{\text{the product of the diameters and numbers of teeth of the driven wheels}}. \qquad (55)$$

An idle wheel such as gear E in Fig. 145 has no effect on the speed ratio, but does cause a change in the direction. This can be seen from the following calculation: Let the wheel D have 100 teeth; E 75 teeth, and F 25 teeth, then the speed of shaft C is equal to the speed of $A\times\frac{100}{75}\times\frac{75}{25}$. 75 then, which is the number of teeth in the idle wheel, appears in both numerator and denominator and cancels out, and therefore the speed of C becomes the speed of $A\times\frac{100}{25}$.

116. Driving and Driven Gears having Coincident Axes. Fig. 146 is a diagram of the back gear arrangement for a simple cone pulley head-stock on an engine lathe. It illustrates the principles involved when two wheels, whose axes coincide, are connected by a train of wheels through an intermediate shaft, the axis of the intermediate shaft being parallel to the axis of the connected wheels. *P* is the cone pulley. *A* is a gear integral with *P* and meshing with gear *B*.

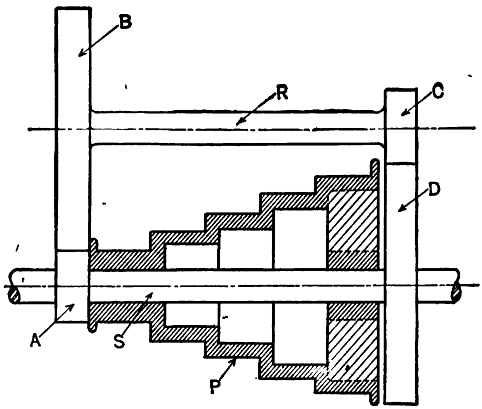


FIG. 146

C is another gear on the same shaft with *B*, both *B* and *C* being fast to the shaft. *C* meshes with gear *D* on the spindle *S*. From Eq. 53,

Speed of spindle = speed of cone pulley

$$\times \frac{\text{Teeth in } A \times \text{Teeth in } C}{\text{Teeth in } B \times \text{Teeth in } D}.$$

Since, however, the shaft *R* is parallel to *S* the gears must be so proportioned that the pitch radius of *A*+pitch radius of *B* equals pitch radius of *C*+pitch radius of *D*. Consequently, if the gears are all of the same pitch, it follows that the sum of the teeth in *A* and *B* must equal the sum of the teeth in *C* and *D*.

117. Screw Cutting in a Lathe. In cutting a screw thread in a lathe, the stock on which the thread is being cut turns at a speed such that it will have a surface velocity suitable for the cutting tool. While the work is making one turn, the tool must be fed along in a direction parallel to the axis of the work a distance equal to the lead of the thread which is being cut. Figs. 147*a* and 147*b* show in a diagrammatic way one of the simplest methods of accomplishing this result. Fig. 147*a* is the front view of the lathe and Fig. 147*b* the end view. The gears are lettered alike in both views.

Many of the modern lathes use a much more elaborate system of gearing, but that shown in the figure serves to illustrate the principles and is easier to understand than the more complicated ones.

In Fig. 147*a*, W is the stock on which the thread is to be cut. This is clamped to the face plate by the dog so that both turn together. The face plate is fast to the spindle which is driven from the cone pulley either directly or through the back gears. On the opposite end of the spindle is the gear A driving gear B on the stud K through one or two idle gears according to the desired direction of rotation. Fast to the same stud, and, therefore, turning at the same speed as B, is the gear C. This gear drives D through an idle gear. D is fast to the lead screw which is embraced by a nut inside the carriage. The tool is supported on and moves along with the carriage.

Assume that the lead of the thread to be cut on the blank is $\frac{1}{n}$ part of an inch and that the lead of the thread on the lead screw is $\frac{1}{t}$ part of an inch. If the blank makes a turns in a unit of time, then the distance which the tool must move in that time must be $a \times \frac{1}{n}$ also; if b represents the number of turns which the lead screw makes in the same unit

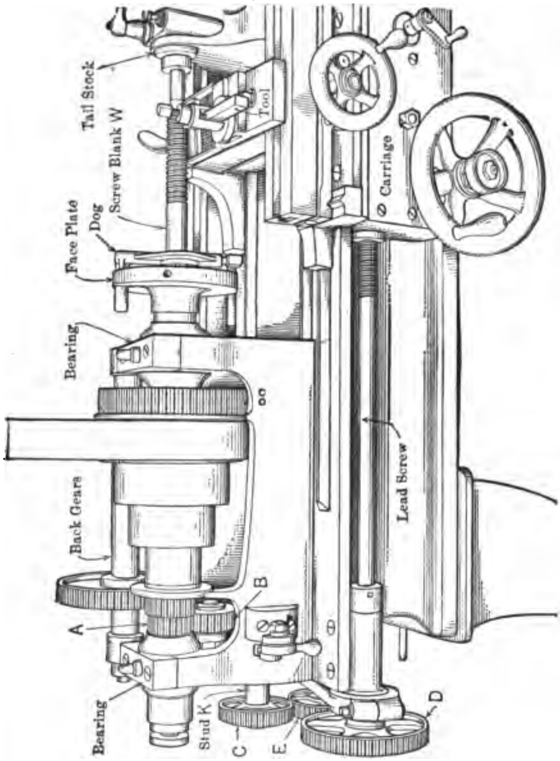


FIG. 147 (a).

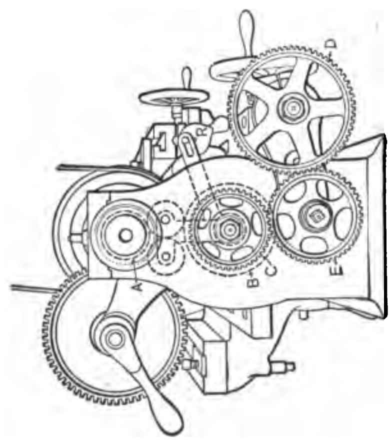


FIG. 147 (b).

of time, $b \times \frac{1}{t}$ must equal the distance the tool moves. Therefore,

$$a \times \frac{1}{n} = b \times \frac{1}{t}.$$

Therefore,

$$\frac{b}{a} = \frac{\frac{1}{n}}{\frac{1}{t}},$$

or

$$\frac{\text{angular speed of lead screw}}{\text{angular speed of blank}} = \frac{\text{lead of thread which is being cut}}{\text{lead of thread on lead screw}}. \quad (56)$$

Now from Eq. (53),

$$\frac{\text{angular speed of lead screw}}{\text{angular speed of blank}} = \frac{\text{teeth in } A}{\text{teeth in } B} \times \frac{\text{teeth in } C}{\text{teeth in } D}.$$

Therefore,

$$\frac{\text{teeth in } A}{\text{teeth in } B} \times \frac{\text{teeth in } C}{\text{teeth in } D} = \frac{\text{lead of thread which is being cut}}{\text{lead of thread on lead screw}}. \quad (57)$$

In any particular lathe the teeth in gears A and B are known quantities and cannot be changed.

The lead of the thread on the lead screw is also known. The gears C and D can be changed to give the desired speed to the lead screw, the idler E being adjusted so as to make proper connection between them. If the thread on the lead screw and that being cut are both right hand or both left hand, the lead screw must turn in the same direction as the blank. If one thread is right hand and the other left hand, the lead screw and the blank must turn in opposite directions.

Example 37. In Fig. 147*a* and 147*b*, assume that the lead of the thread on the lead screw is $\frac{3}{8}$ in. left hand; gear A has 20 teeth; B, 30 teeth; C, 27 teeth, and D, 54 teeth.

To find the lead of the thread which is being cut on the blank.

Solution. Substituting in Eq. (57),

$$\frac{20}{30}\times\frac{27}{54}=\frac{\text{lead of thread being cut}}{\frac{3}{8}}.$$

Solving this equation gives lead of thread which is being cut as $\frac{1}{8}$ in. That is a screw of 8 threads per inch is being cut.

To determine whether a right-hand or a left-hand thread is being cut the directions may be followed through by putting on arrows, as shown. In this case the arrows indicate that the blank and the lead screw are turning in the same direction, therefore, since the lead screw has a left-hand thread the thread which is being cut is left hand. If the lever R were thrown up so as to bring both idle gears into use, the direction of the lead screw would be reversed and a right-hand thread would be produced.

Example 38. Referring still to Figs. 147*a* and 147*b*, assume that the lead screw and the gears A and B are the same as in Example 37. Let it be required to find the number of teeth in C and D to cut 20 threads per inch on the blank.

Solution. Substituting in Eq. (57),

$$\frac{20}{30}\times\frac{\text{Teeth in } C}{\text{Teeth in } D}=\frac{\frac{1}{20}}{\frac{3}{8}}.$$

Hence,

$$\frac{\text{Teeth in } C}{\text{Teeth in } D}=\frac{1}{20}\times\frac{8}{3}\times\frac{30}{20}=\frac{1}{5}.$$

Then any practical sized gears may be used at C and D provided D has five times as many teeth as C; as for example, 20 teeth in D and 100 teeth in C.

118. Designing Gear Trains. No definite rules or formulas can be followed in designing a train of gears to have a certain train value. The process is mainly one of " cut and try " until the desired result is obtained. There are, how-

ever, some general lines of attack which may be followed, and which may best be understood by studying certain typical problems. If the value of the train is chosen arbitrarily, it may be found impossible to select gear which will give exactly the value called for.

Example 39. Let it be required to select the gears for a train in which the last gear shall turn 19 times while the first gear turns once. No gear to have less than 12 teeth nor more than 60 teeth.

Solution. The first step in the solution of this problem is the determination of the number of pairs of gears necessary to give a train value of 19 and keep the gears within the limits of size specified. If only one pair were used, making the

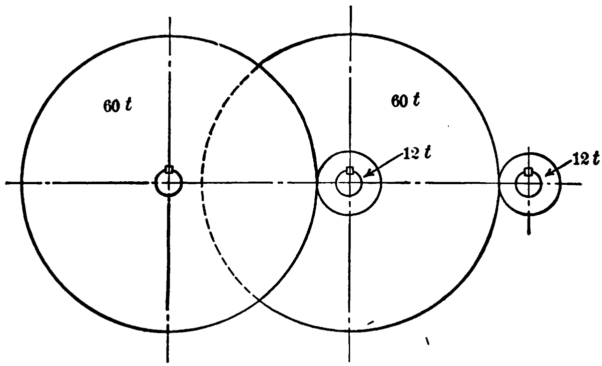


FIG. 148.

driver as large as allowed, that is, 60 teeth, and the driven gear as small as allowed, that is 12 teeth, the train value would be $\frac{60}{12}$ or 5. If a second driver of 60 teeth is made fast to the 12 teeth gear, and this drives a second gear of 12 teeth, as shown in Fig. 148, the train value becomes $\frac{60}{12}\times\frac{60}{12}=25$. This is greater than the assigned value of 19, therefore, two pairs of gears will be sufficient to obtain a value of 19 without exceeding the specified limits of size. Having thus determined the number of gears necessary,

the next step is the selection of the gears themselves to give the exact value of 19. This may be tried in several ways. First, since two pairs of gears are to be used, the square

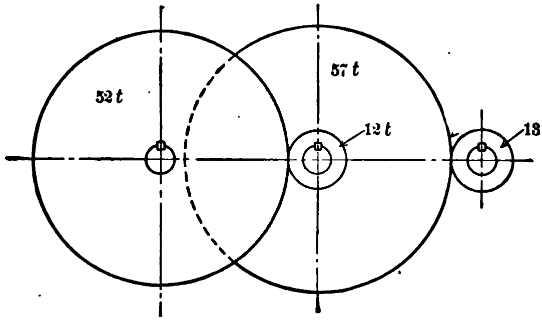


FIG. 149.

root of 19 may be taken. This is approximately 4.36. This does not differ greatly from $4\frac{1}{3}$. Now

$$\frac{4\frac{1}{3}}{1}\times\frac{19}{4\frac{1}{3}}=19.$$

Multiply both numerator and denominator of the first fraction by 12, and of the second fraction by 3 and the equation becomes

$$\frac{52}{12}\times\frac{57}{13}=19.$$

Therefore, a train of gears as shown in Fig. 149 will give the required train value of 19.

Example 40. Let it be required to find suitable gears to drive a shaft at 4 r.p.m. from a shaft 500 r.p.m., using no gear of less than 24 teeth nor more than 96 teeth.

Solution. The train value in this case is $\frac{4}{500}=\frac{1}{125}$.

Since the smallest gear that can be used is 24 teeth and the largest 96 teeth, the train value between any gear and its driver cannot be less than $\frac{24}{96}$ or $\frac{1}{4}$.

Therefore $\frac{1}{4}$ must be multiplied by itself a sufficient number of times to obtain a final result less than $\frac{1}{125}$. Now,

$(\frac{1}{4})^3 = \frac{1}{64}$; then 3 pairs of gears will not be enough. But $(\frac{1}{4})^4 = \frac{1}{256}$, which is beyond the specified $\frac{1}{125}$, hence 4 pairs will be proper to use.

The fourth root of $\frac{1}{125}$ is approximately $\frac{1}{3\frac{1}{3}}$ or $\frac{3}{10}$.

A train of gears for a trial might be in the ratio of

$$\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{\frac{1}{125}}{(\frac{3}{10})^3} = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{8}{27}.$$

Multiplying numerator and denominator of each of the first three terms by 9 and of the last term by 3 gives

$$\frac{27}{90} \times \frac{27}{90} \times \frac{27}{90} \times \frac{24}{81}.$$

Then a train, such as shown in Fig. 150, fulfils the required conditions.

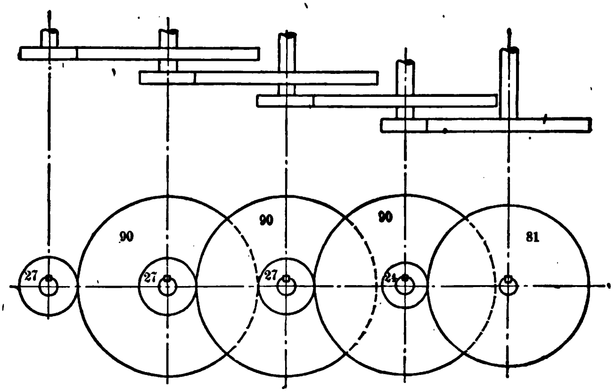


FIG. 150.

119. Epicylic Trains. An epicylic train of gears is a train in which some of the gears turn on fixed axes, while other of the gears are on axes which are themselves in motion. Fig. 151 illustrates this.

B is a gear turning with the shaft *S* which is in stationary bearings and is driven by the gears *R* and *K*. *C* is a gear meshing with *B*. The stud *T* on which *C* turns is carried

by the arm A. Fast to the hub of A is the gear E driven by the gear D. Attached to C is the gear F which drives G. G turns on the same axis as B, but must, of course, be free to

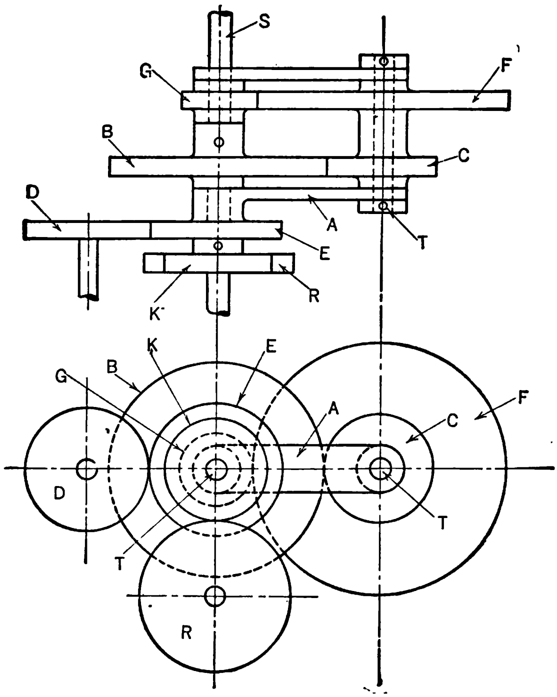


FIG. 151.

turn at a different speed from B. D and R receive their motion from outside sources.

If the gear D is first assumed not to turn, the arm A will be stationary, and the following equation will hold true:

$$\frac{\text{Speed of } G}{\text{Speed of } B}=\frac{\text{Teeth in } B\times\text{Teeth in } F}{\text{Teeth in } C\times\text{Teeth in } G},$$

or speed of G=speed of $B\times$train value.

If, on the other hand, the gear B is assumed not to turn and D turns at a definite speed, the arm A will revolve, the stud T will travel around S as an axis, C rolling around on B, and F rolling around on G. This will impart motion to G which may be found as follows: Suppose A to have a speed of a r.p.m. left handed; then the speed of C on its own axis is the same as if A were held still and B turned with a r.p.m. right-handed. That is, C would have a speed on its axis of $a\times\dfrac{\text{Teeth in } B}{\text{Teeth in } C}$ r.p.m. left-handed, relative to its own axis, or relative to the arm.

This speed of C would impart to G, relative to the arm, a speed of $a\times\dfrac{\text{Teeth in } B\times\text{Teeth in } F}{\text{Teeth in } C\times\text{Teeth in } G}$ r.p.m. $=a\times$Train value, right-handed. But the arm is itself turning left-handed at a speed a r.p.m. Therefore, the actual speed of G due to the speed of A is $a-a\times$train value. The resultant speed of G, due to the combined speeds of B and A, is the algebraic sum of the speeds which it would have when each moved with the other standing still, or

$$\begin{aligned}\text{Speed of } G=\text{Speed of } B\times \text{ Train Value}+\text{Speed of Arm}\\ -\text{Speed of Arm}\times\text{ Train Value.} \quad . \quad . \quad . \quad (58)\end{aligned}$$

This may be expressed by letters, thus,

$$n=me+a-ae, \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where $n=$the speed of the last wheel of an epicyclic train, m the speed of the first wheel of the train, a the speed of the arm and e the train value.

120. Absolute and Relative Speed. Assume that C, Fig. 152, is a gear carried by the arm A and pinned to A so that it cannot turn on its own axis. If A is caused to turn about S once, a reference mark V on C, which in the position shown is pointing downward, would point in every direction successively as A revolved and come finally to its present position when the arm had made a complete turn.

If C is meshing with another gear as in the case of the gears C and B in Fig. 151, and therefore rolls on its axis at the same time that the axis revolves, the reference mark would swing around and point to its original direction a number of times equal to the algebraic sum of the speed of the arm and the speed of C on its axis. This resultant number of times that the reference mark returns to its original direction, having in the meantime turned completely

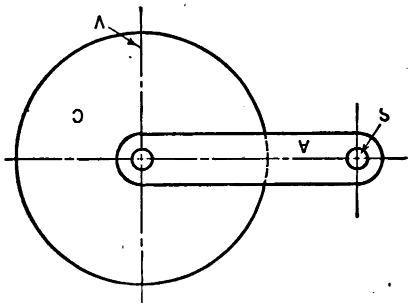


FIG. 152.

over, is called the number of *absolute terms* that C makes, or its *absolute speed.* The speed of C on its own axis is called its *relative speed.*

121. Solution of Problems on Epicylic Trains. Problems relating to epicyclic trains may be solved by the formula given in Eq. (59) or by a method sometimes called the *tabulation method.*

Example 41. In Fig. 151 let B have 80 teeth, C 40 teeth, F 90 teeth and G 30 teeth. If B has a speed of 100 r.p.m. right-handed and A 60 r.p.m. left-handed, let it be required to find the speed of G.

Solution No. 1. Using Eq. (59) let right-handed rotation be assumed plus. Then $m=+100$, $a=-60$, $e=\frac{80}{40}\times\frac{90}{30}=6$, and since with the arm at rest G would turn in the same direction as B, the value of e is plus. That is $e=+6$.

Substituting in Eq. (59),

$$n=100\times 6+(-60)-(-60\times 6)=600-60+360=900.$$

Solution No. 2. Using the tabulation method, the motions of B and of the arm are assumed to take place successively instead of simultaneously, the gears are first assumed to be made fast to the arm so that there can be no relative motion. The arm is made to turn at the proper speed for a unit of time in the proper direction; all the gears will turn with it. Then the arm is held still and the first gear (in this case, B) is turned backward or forward enough to make its net number of turns equal to its known speed. These steps are tabulated as follows:

	Arm	B	C
Gears locked to arm..	-60	-60	-60
Arm held still........	0	$+160$	160×6
	-60	$+100$	$960-60$ $=900$

Example 42. In Fig. 153 let gear B have 24 teeth and C 18 teeth. If B is held from turning and the arm makes 1

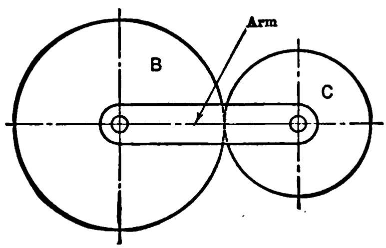


FIG. 153.

turn right-handed, let it be required to find how many absolute turns C makes. (See § 118.)

Solution No. 1. Using Eq. 59, $m=o$, $e=-\frac{24}{18}=-\frac{4}{3}$, $a=+1$ (assuming right-hand rotation is plus). Then, substituting in Eq. (59), $n=o+1-(-\frac{4}{3})$, or, $n=\frac{7}{3}$.

Solution No. 2. Using the tabular method:

	Arm	B	C
Gears locked to arm. .	$+1$	$+1$	$+1$
Arm held still.	0	-1	$-1(-\frac{4}{3})$
			$+\frac{7}{3}$

Example 43. In Fig. 154 E is an annular gear which cannot turn, being fast to the frame of the machine. The

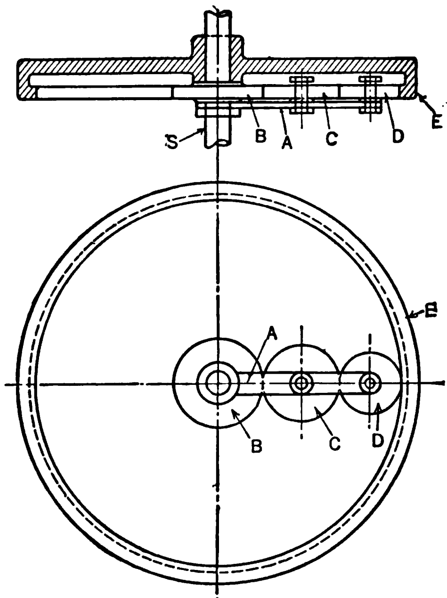


FIG. 154.

arm A turns about the shaft S which is also the axis of the gears B and E. B has 24 teeth, C 20 teeth, D 16 teeth, and E 96 teeth. Let it be required to find the speed of the gear B to cause the arm A to have a speed of 25 r.p.m. right-handed.

Solution. Assume B to be the first wheel of the train and assume right-handed rotation as $+$:

Then referring to Eq. (59) $n=o$, $a=+25$, $e=+\frac{24}{96}=+\frac{1}{4}$.

Substituting these values in the equation,

$$O=m\times\frac{1}{4}+25-25\times\frac{1}{4},$$

whence

$$m=-75.$$

Therefore, B will have to have a speed of 75 r.p.m. left-handed to give the required speed to the arm A.

Example 44. In Fig. 155, B and E are two bevel gears running on shaft S, but not fast to it. Attached to the collar

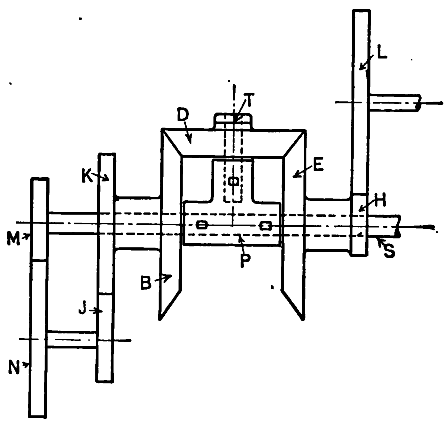


FIG. 155.

P, which is set screwed and keyed to S, is a stud T on which turns freely the gear D meshing with B and E. B and E are of the same size and T is at right angles with S. J is a gear having 25 teeth and driving the 40-tooth gear K which is fast to B. L is a 51-tooth gear driven by the 17-tooth gear H which is fast to E. It is required to find the speed of L if J makes 40 r.p.m. N is a 45-tooth gear fast to the same shaft as J and drives the 20-tooth gear M which is fast to S.

Solution. The first step is to pick out those gears which are a part of the epicyclic train. These are evidently B, D, and E. The epicyclic arm is T. Assume B as the first wheel of the epicyclic train, E the last wheel, and, letting m represent the speed of B, n the speed of E, a the speed of S and e the train value between E and B. Also assume direction in which J turns as positive. Using Eq. (59),

$$e = -1,$$

$$m = -\tfrac{25}{40} \times 40 \text{ r.p.m.} = -25 \text{ r.p.m.},$$

$$a = -\tfrac{45}{20} \times 40 = -90.$$

Then, substituting in Eq. (59),

$$\begin{aligned} n &= (-25) \times (-1) + (-90) - \{(-90) \times (-1)\} \\ &= 25 - 90 - 90 \\ &= -155 \text{ r.p.m.} = \text{speed of } E. \end{aligned}$$

$$\text{Speed of } L = -155 \times (-\tfrac{17}{51}) = 51\tfrac{2}{3}.$$

Therefore, L has a speed of $51\frac{2}{3}$ r.p.m. in the same direction as J.

Example 45. *Automobile Transmission.* Fig. 156 shows the arrangement of gears in the differential of an automobile. Shaft S is driven from the motor and has keyed to it the bevel gear D meshing with E which turns loosely on the hub of the gear H, the latter being keyed to the axle of the left wheel. E has projections on it which carry the studs T furnishing bearings for the gears R. There are several of these gears in order to distribute the load. The gears R mesh with G which is, as has been said, fast to the axle of the left wheel, and with K which is fast to the axle of the right wheel. When the automobile is going straight ahead D drives E and all the other gears revolve as a unit with E without any relative motion. As soon, however, as the car

starts to turn a corner, say toward the right, the left wheel will have to travel further, and therefore the shaft B must turn faster than C. Then the gears begin to move relative to each other, the action being that of an epicyclic train.

Example 46. *Triplex Pulley Block.* Fig. 157 shows a vertical section and side view with part of the casing removed,

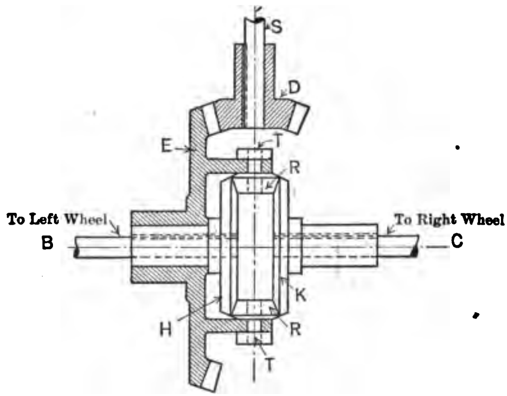


FIG. 156.

of a triplex pulley block. S is the shaft to which the hand chain wheel A is keyed. Also keyed to S is the gear F meshing with the two gears E. The gears E turn on studs T which are carried by the arm B, the latter being keyed to the hub of the load chain wheel G. The gears C are integral with E and mesh with the annular D which is a part of the stationary casing. The mechanism is an epicyclic train. F is the first wheel of the train and has a speed imparted to it by the turning of the hand chain wheel A. The annular D is the last wheel of the train and does not turn. The train value is

$$\frac{\text{Teeth in } F}{\text{Teeth in } E} \times \frac{\text{Teeth in } C}{\text{Teeth in } D},$$

and is negative. Assuming one turn of A, the turns of the

arm B may be found, and, therefore, the turns of G. Hence, knowing the angular speed of A and its diameter, and the angular speed of G and its diameter, the relative linear

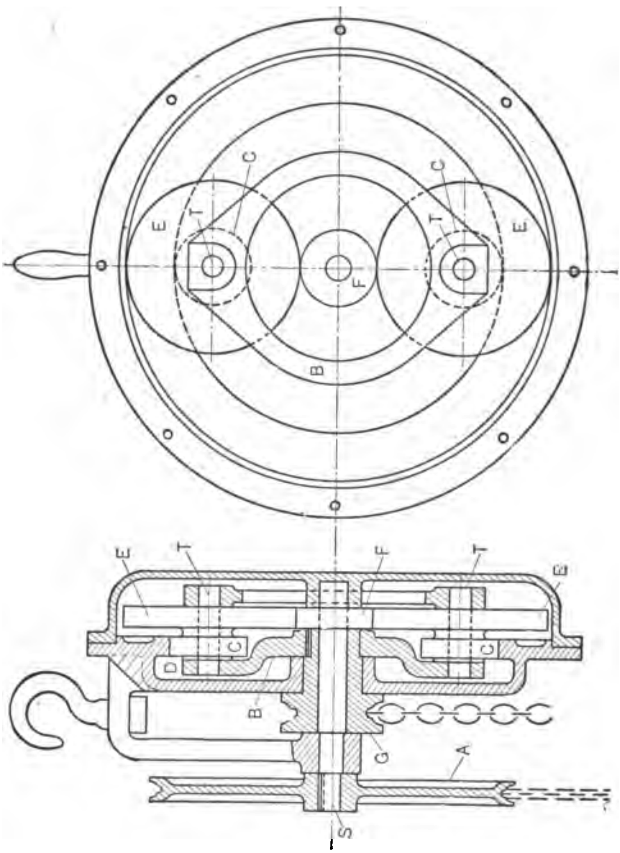


FIG. 157.

speeds of the hand chain and the load chain can be calculated. The load will then be to the force exerted on the hand chain as the speed of the hand chain is to the speed of the load chain, friction being neglected.

122. Pulley Blocks. The ordinary pulley blocks, while differing from both the simple wheel trains and the epicyclic trains, partake somewhat of the characteristics of both and can logically be discussed at this point.

The *mechanical advantage* of a hoist is the ratio of the weight which can be lifted to the force which is exerted, friction being neglected.

The action of pulley block hoists and the method of calculating the weight which can be lifted by a known force is best illustrated by the solution of a number of examples.

Example 47. *Hoist with Two Single Sheave Blocks.* In Fig. 158 the upper block A, known as the standing block, is suspended from a fixed support. The rope is made fast to the casing of the upper block, passes around the sheave in the lower block and up around the sheave P which turns about the axis S in the upper block. It is required to find the force at F necessary to raise a weight W of 100 lbs. suspended from the lower block.

Solution. From the principles discussed in § 12 it can be seen that the force at F is to the weight W as the linear speed of W is to the linear speed of the rope at F. The problem then becomes one of finding the speed ratio of W and F. Assume that W is lifted 1 ft. by some external force with the rope at F not moving. Then 1 ft. of slack rope would result at R and another foot of slack at T, giving a total of 2 ft. of slack which must be drawn over to F in order to keep the rope tight. Therefore, the linear speed of F is to the linear speed of W as 2 is to 1. Hence, F is to W as 1 is to 2, or $F=\frac{1}{2}W=50$ lb.

Example 48. *Hoist with One Single Block and On Double Block.* The hoist shown in Fig. 159 has the part of the rope which is marked T made fast to the lower block; it then passes over a sheave in the upper block, comes down at R and passes under the sheave in the lower block, then up at P over a second sheave in the upper block and off at F.

It is required to find the mechanical advantage of this hoist; that is, the ratio of the weight W to the force at F.

Solution. Applying the same method used in example 47, shows 1 ft. of slack in each of the three parts R, T, and P,

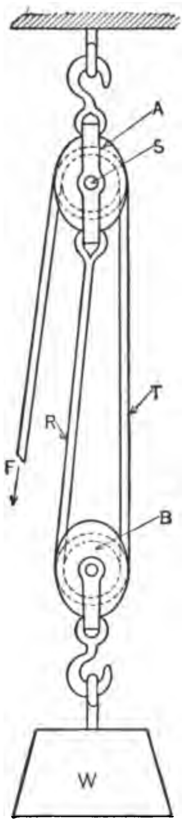


FIG 158.

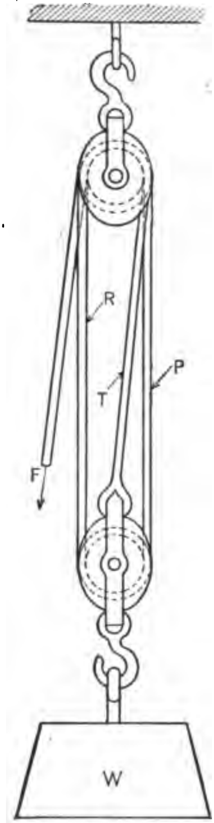


FIG. 159.

or a total of 3 ft. which must be drawn off at F if W is lifted 1 ft. by an external force. Therefore,

$$\frac{W}{F}=\frac{3}{1}.$$

Example 49. "*Luff on Luff.*" Fig. 160 shows a combination of two sets of pulley blocks, the rope F of the first set being made fast to the moving block of the second set.

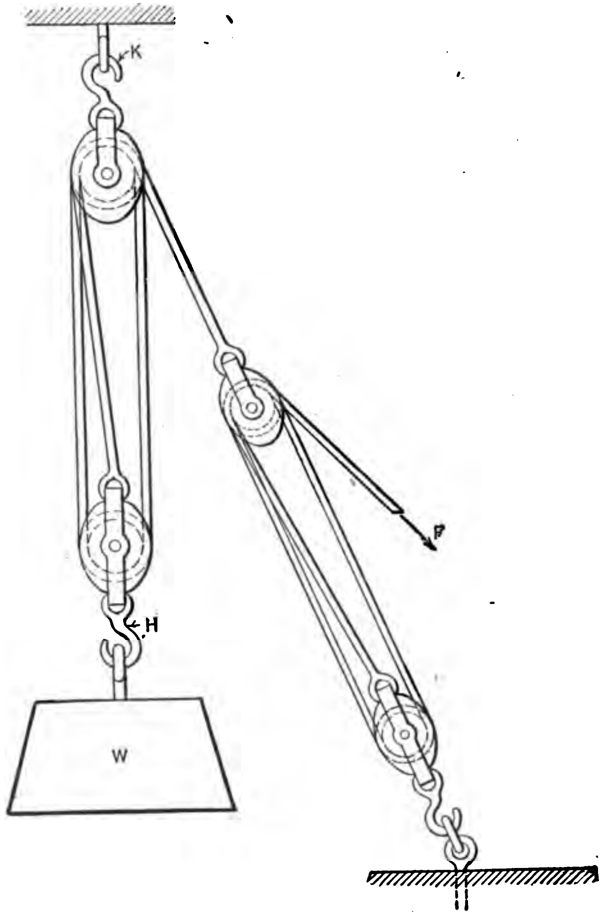


FIG. 160.

Solution. The mechanical advantage of each set is found as in the previous examples. Then the product of

the two will be the mechanical advantage of the combination. The first set in this case has a mechanical advantage of 3, and the second set of 4; therefore the combination has a mechanical advantage of 12. If the hook H were attached to a stationary support and the load applied to the hook K, the advantage of the system would be 16.

Example 50. "*Spanish Burton.*" If the weight W (Fig. 161) is lifted 1 ft., a foot of slack is caused at both P and R.

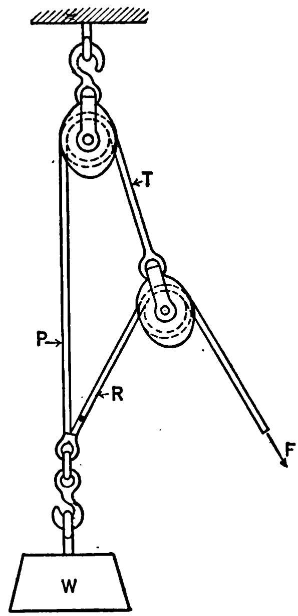


FIG. 161.

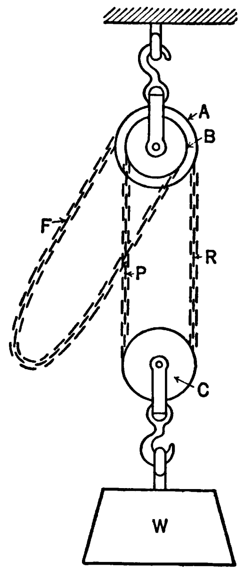


FIG. 162.

The foot at P is carried over to T which, in turn, causes a foot of slack in both R and F; this makes a total of 2 ft. of slack in R which must be drawn over to F in addition to the 1 ft. already given to F from T. Therefore, 3 ft. must be taken up at F for every foot that W is lifted. Then the mechanical advantage is 3.

123. Weston Differential Pulley Block. Fig. 162 shows a chain hoist known as the Weston Differential Pulley Block. The two upper sheaves A and B are fast to each other. The diameter of A is a little larger than the diameter of B and it is the ratio of these two diameters which governs the mechanical advantage.

The diameter of the lower sheave C is usually a mean between the diameters of the upper ones in order that the supporting chain may hang vertically. This feature is not of great importance, and the diameter of the lower sheave has no effect on the mechanical advantage.

The chain is endless, passing over A, down at R, under C, up at P, around B, and hanging loose. The lifting force is applied at F. The sheaves are so shaped that the links of the chain fit into spaces provided for them to prevent slipping.

The operation of the hoist may be seen from the following:

Let D_a represent the pitch diameter of the sheave A, D_b the pitch diameter of the sheave B. Assume that the chain is drawn down at F fast enough to cause A to make one complete turn in a unit of time; that is, F has a speed of πD_a linear units. This would give an upward speed to the chain at R of πD_a linear units. Then, if B were not turning the sheave C would roll up on P, its center rising at a speed equal to one-half the speed of the chain at R; that is, the center of the lower sheave, and therefore the weight W, would rise at a speed of $\frac{\pi D_a}{2}$ linear units. But, at the same time that R is rolling C up on P the pulley B is turning at the same angular speed as A, and therefore paying out chain at P at the rate of πD_b linear units per unit of time. This causes C to roll down on R at a speed such that its center is lowered at a speed of $\frac{\pi D_b}{2}$ linear units. The resultant upward speed of the center of C is, therefore,

$$\frac{\pi D_a}{2}-\frac{\pi D_b}{2}=\frac{\pi(D_a-D_b)}{2}.$$

Since the speed of F is πD_a the ratio of the speed of F to that of W is

$$\frac{\pi D_a}{\frac{\pi(D_a-D_b)}{2}}=\frac{2D_a}{D_a-D_b}.$$

CHAPTER IX

LINKS AND LINKAGE

124. Four-bar Linkage. Fig. 163 shows in a simple form a mechanism known as a four-bar linkage. *E* is a fixed piece, such as the frame of a machine. *A* and *D* are shafts having their bearings in *E*. The piece *F*, called a *crank*, is keyed to *A*. *H* is a similar crank keyed to *D*. The outer

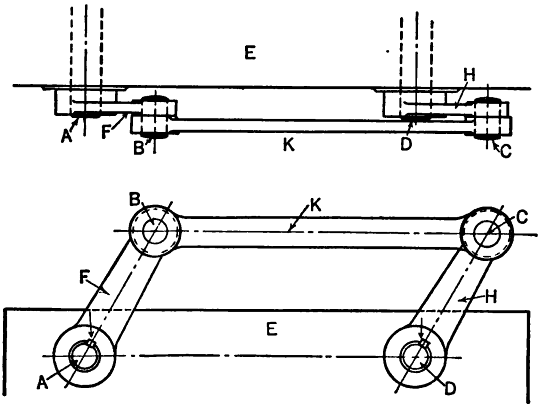


FIG. 163.

ends of *F* and *H* are connected to each other by the *connecting rod* *K* and the *crank pins* *B* and *C*. *B* may be made fast to *K* and be free to turn the hole in *F* or it may be fast to *F* and free to turn in *K*. Similarly, the pin *C* may be free to turn in either *H* or *K*.

If shaft *A* is caused to revolve, the crank *F* will revolve with it, the center of the pin *B* moving in a circle whose

center is the center of A. This movement of B will, through the connecting rod K, cause the pin C to move, and since C can move only in a circle about the center of D the crank H will be caused to turn, turning D with it. Each one of the pieces E, F, K, and H is called a *link*, and the whole system is called a *four-bar linkage.*

It is convenient, in studying linkages, to indicate them by the center lines of the links, as shown in Fig. 164, which

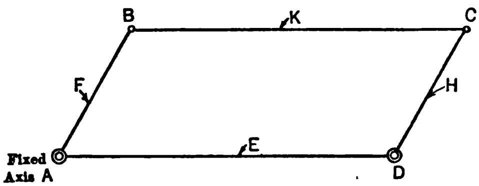


FIG. 164.

represents the same linkage as that shown in Fig. 163. The line joining the centers of the two stationary axes A and D is called the *line of centers,* the center lines AB and DC joining the stationary axes to the centers of the crank pins are called the *center lines of the cranks,* and the line joining the centers of the crank pins B and C is called the *center line of the connecting rod* or *line of connection.*

125. Relative Motions of the Links. In the four-bar linkage shown in Fig. 165, A and D are the stationary axis, AB and DC the cranks and BC the connecting rod. If the crank AB turns from the position shown in full lines to the right-hand dotted position—that is, turns through the angle BAB_1—the pin B will travel over the arc BB_1. This will cause the connecting rod to move and push the pin C along its path to C_1.

It is apparent from the figure that the length of the arc CC_1 is not equal to the length of the arc BB_1. In other words, with the several links having the relative lengths as here shown, the linear speeds of the crank pins will differ.

If, furthermore, B is moved to B_2, C will move to C_2. Now the arc BB_2 is made equal to BB_1, but CC_2 is evidently not equal to CC_1. This construction, therefore, suggests that if the crank AB is turned with uniform angular speed so that

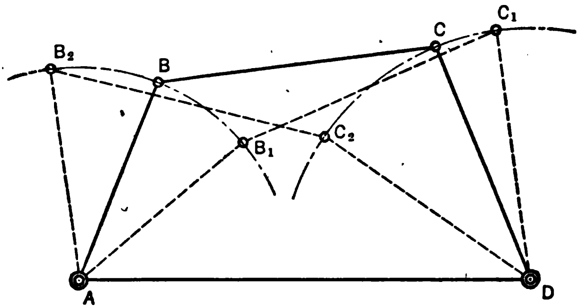


Fig. 165.

the crank pin B has a uniform linear speed, the crank pin C has a varying linear speed and the crank DC a varying angular speed. It should be observed that the motions of the links of a specific four-bar linkage *relative to each other* are always the same whichever of the four links is the stationary one.

It is the purpose of the present chapter to discuss methods of determining the relative linear speeds of different points in four-bar linkages and also the relative angular speeds of the links.

126. Graphical Representation of Linear Velocities. For convenience in graphical work in connection with the study of velocities it is customary to represent the velocity of a point by a straight line whose direction and length indicate the direction and magnitude of the velocity of the point. For example, let it be assumed that the block M (Fig. 166) is sliding to the right on the guide, at the rate of 1 ft. per second. If it is desired to represent the velocity of any point A on this block by a line, any unit of length may be

assumed to represent 1 ft. Suppose 1 in. to the foot is chosen as the convenient scale, then a line 1 in. long is drawn from A toward the right with an arrowhead at its end pointing to the right. If, now, another block N is

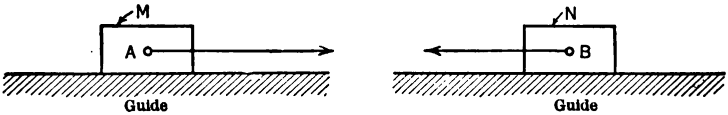


FIG. 166.

moving to the left at a speed of $\frac{3}{4}$ in. per second, the velocity of any point B on this block will be indicated (if the scale is to be the same as that used for A) by a line $\frac{3}{4}$ in. long pointing to the left.

127. Direction of Velocity of a Point Moving in a Curved Path. A point which is moving in a straight line has a velocity whose direction remains unchanged so long as the point continues to move along that line without returning over the same path. A point on a body which is moving

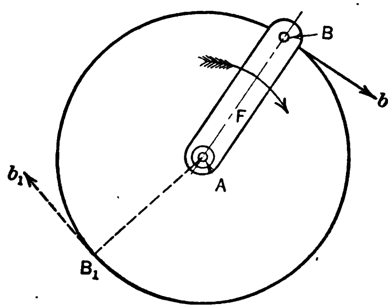


FIG. 167.

over a curved path has a velocity whose direction is constantly changing.

At any given instant, the direction of the velocity of a point which is moving along a curved path is tangent to the curve

at the place where the moving point is at the instant in question. This is true because if the force which constrains the body to move in the curved path be suddenly removed the body, and therefore every point on the body will continue to move in a straight line tangent to the curve. This principle is roughly illustrated by whirling a stone attached to a string and suddenly letting go of the string.

In Fig. 167, F is a crank revolving right-handed, A being the axis and B the center of the crank pin. The point B at this instant is moving in the direction indicated by the line Bb drawn perpendicular to AB. When the crank has turned so that its center line AB occupies the position AB_1 the direction of the center of the crank pin is indicated by the line B_1b_1 drawn perpendicular to AB_1. The length of Bb or B_1b_1 may be such as to indicate the magnitude of the velocity, that is the linear speed, the scale being any convenient one.

128. Relative Linear Velocities of the Crank Pins in a Four-bar Linkage. Assuming that the crank AB (Fig. 168) is turning at such an angular speed that the crank pin B has a linear velocity represented by the line Bb, let it be required to find the line which represents, at the same scale, the linear velocity of C. Since Bb indicates the velocity of B at the instant, B would, if not restrained by the crank, move to b in a unit of time. The same point would be reached if for a part of the unit of time B should have the velocity represented by Be, moving along the line CB extended, to the point e which is the foot of a perpendicular let fall from b to CB; then from e moving out along this perpendicular with a velocity represented by eb. That is, the actual velocity of B may be considered as the *resultant* of two other velocities, one in the direction of the line CB and the other at right angles to that line. These two velocities, which are considered as composing the actual velocity, are called *components* of the actual velocity.

Although B does not actually go to b its tendency at the

instant is to go there in a unit of time, and its tendency to move along the line CB is such that, if there were no components at right angles to CB, it would go to e in a unit time. It is this tendency of B to move along the line CB, that is, the component of Bb along the line CB which causes C to move, and since CB is a rigid rod which can be neither

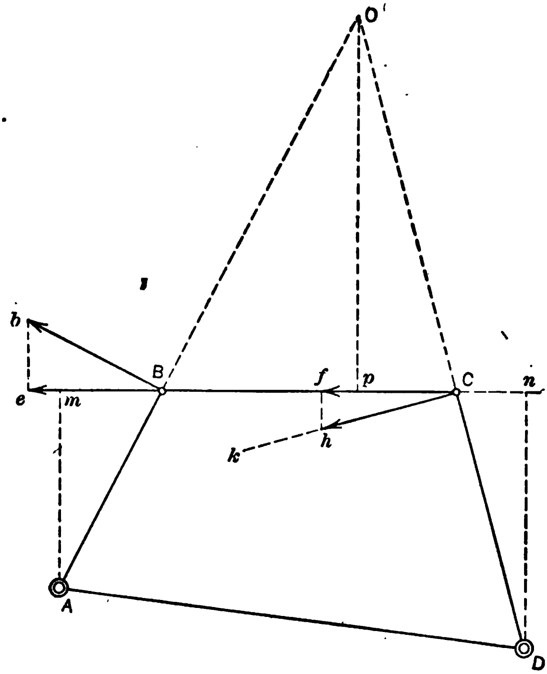


FIG. 168.

lengthened nor shortened, C must have a component velocity (or tendency to move) along the line CB equal to that of B, that is, equal to Be. The actual direction of the velocity of C is along a line Dk perpendicular to DC. Then, to find the velocity of C lay off Cf equal to Be and through f draw a line perpendicular to CB meeting Ck at h. Ch then rep-

resents the velocity of C at the same scale at which the velocity of B is represented. For example, if Ch is found to be three-fourths as long as Bb, it would indicate that at that instant C has a velocity which is three-fourths of that of B.

129. Instantaneous Axis of Connecting Rod. The discussion in the preceding paragraph shows that one end of the connecting rod in Fig. 168 is moving at the instant in the direction Bb, perpendicular to AB, while the other end is moving in the direction Ch, perpendicular to DC. The direction of B would be the same wherever the axis of rotation were located on the line AB or AB extended. Similarly, the direction of C would be the same wherever its axis of rotation were located on DC or DC extended. Hence, both these points B and C might be treated for the instant as if they were turning about the point which is common to AB and DC, that is, their point of intersection O. B and C are points on the connecting rod BC, as well as on the cranks, therefore, the whole rod BC may be treated for the instant as if it were turning about O as an axis. The point O is called the *instantaneous axis* of BC. The principle is the same as if the connecting rod were made fast to a wheel which, for the moment, is turning about O as an axis. It follows that the velocity of C is to the velocity of B as OC is to OB.

This law may be stated thus:

The linear velocities of any points on the connecting rod of a four-bar linkage are to each other as their respective distances from the instantaneous axis of the rod. . . . (60)

130. Centroid. If the position of the instantaneous axis of a connecting rod or other moving link is found for a series of positions through the entire cycle of motion of the linkage, and a smooth curve is drawn through these different positions of the instantaneous axis, the curve thus found is called the *centroid* of the link in question. Some of the properties of centroids and the uses made of them will be taken up later

in connection with certain special examples of four-bar linkages.

131. Relative Angular Speeds of the Cranks. Referring to Fig. 168, the angular speed of AB expressed in radians is equal to the linear speed of $B \div$ the length of AB (see Eq. 3). Therefore, since Bb represents the linear speed of B,

$$\text{Angular speed of } AB = \frac{Bb}{AB}.$$

Similarly,

$$\text{Angular speed of } DC = \frac{Ch}{DC},$$

or

$$\frac{\text{Angular speed of } DC}{\text{Angular speed of } AB} = \frac{\frac{Ch}{DC}}{\frac{Bb}{AB}} = \frac{Ch}{Bb} \times \frac{AB}{DC}. \qquad (61)$$

Now triangle eBb is similar to triangle mAB since AB is perpendicular to Bb and Am is perpendicular to Be. Therefore,

$$\frac{Be}{Bb} = \frac{Am}{AB},$$

or

$$Bb = Be \times \frac{AB}{Am}.$$

Similarly in the triangles fCh and nDC,

$$\frac{Cf}{Ch} = \frac{Dn}{DC},$$

or

$$Ch = Cf \times \frac{DC}{Dn}.$$

Substituting these values for Bb and Ch in Eq. 61 gives

$$\frac{\text{Angular speed of } DC}{\text{Angular speed of } AB} = \frac{Cf \times \frac{DC}{Dn}}{Be \times \frac{AB}{Am}} \times \frac{AB}{DC} = \frac{Cf}{Be} \times \frac{DC}{AB} \times \frac{Am}{Dn} \times \frac{AB}{DC},$$

but Cf is equal to Be by construction, therefore the equation becomes

$$\frac{\text{Angular speed of } DC}{\text{Angular speed of } AB} = \frac{Am}{AB}. \quad . \quad . \quad . \quad . \quad . \quad (62)$$

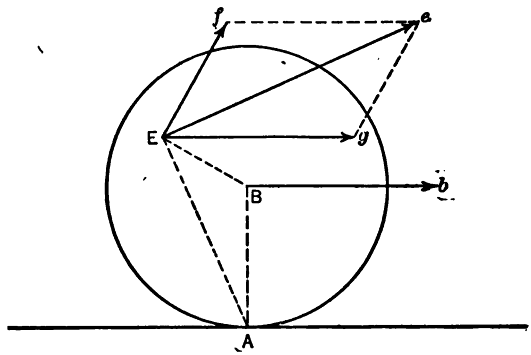


FIG. 169.

This law may be stated in words as follows:

The angular speeds of the cranks of a four-bar linkage are to each other inversely as the lengths of the perpendiculars from the axes of the cranks to the center line of the connecting rod. (63)

132. Wheel Rolling on a Track. Let B, Fig. 169 be the center of a wheel resting on a track at the point A. If a pull is exerted at the center along the line Bb, parallel to the track, B will move in the direction Bb. The effect is the same for the instant as if AB were a crank turning about A as an axis and since AB is an imaginary line on the wheel, the whole wheel becomes a crank turning about A as an axis. In other words:

Any body, which is rolling on a stationary body, has for an instantaneous axis the point of contact between the two bodies. (64)

Any other point on the wheel, as E, must at the instant in question be moving about A as an axis, and therefore

the direction of the velocity of E is Ee perpendicular to the line AE. The magnitude of this velocity is to the magnitude of the velocity of B as AE is to AB.

This velocity Ee is made up of rotation about the center B of the wheel combined with a velocity parallel to the track equal to the velocity of B. That is, the real velocity Ee may be considered as consisting of the components Eg (which is equal and parallel to Bb) and Ef perpendicular to BE and of such magnitude that the figure $gefE$ is a parallelogram.

133. Typical Problems Dealing with Velocities. The fundamental principles governing the relative velocities of points on four-bar linkages and other moving bodies have been outlined in the preceding paragraphs. The only way, however, to understand these principles clearly and to become proficient in applying them to actual mechanisms is to study carefully a number of examples illustrating the different cases. Some of these examples involve special applications of the four-bar linkage which will be analyzed later, but are introduced here merely as examples in velocities, and no attention need be given at this time to their connection with the four-bar linkage.

Example 51. In Fig. 170 let AB and DC be the center lines of the cranks of a four-bar linkage, A and D being the fixed axes. Let the linear velocity of the crank pin B be represented by the line Bb perpendicular to AB. It is required to find the lines which shall represent at the same scale, the linear velocities of the crank pin C and of any point E on the center line of the connecting rod.

Solution. Resolve the velocity Bb into the two components Be and Br along CB and at right angles to CB, respectively (see § 128). Make Cf equal to Be and through f draw a line perpendicular to CB meeting, at h, a perpendicular to DC, drawn through C. Then Ch represents the velocity of C.

To find the velocity of E it is necessary to find, first, the

direction in which E is moving at the instant. This is done by finding the instantaneous axis of CB (see § 129). AB and DC are produced until they meet at O, which is, therefore, the instantaneous axis of CB. OE is next drawn, and

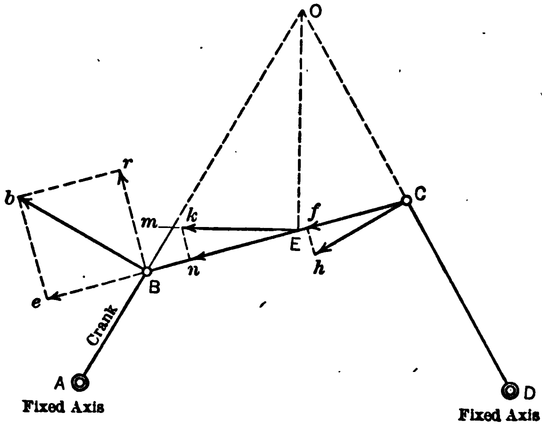


FIG. 170.

the direction of the velocity E is along a line Em perpendicular to OE. The magnitude of this velocity is found by laying off En equal to Be and drawing a line through n perpendicular to CB meeting Em at K. EK is the required line representing the velocity of E.

Example 52. Fig. 171 shows in a diagrammatic form the crank, connecting rod and crosshead of a steam engine. Let Bb represent the velocity of B. It is required to find the velocity of the center C of the crosshead pin and of a point E on the center line of the connecting rod.

Solution. Bb is resolved into components along and at right angles to CB giving Be as the component along CB. Make Cf equal to Be. Now C is moving along the line NM parallel to the guides. Therefore, the velocity of C is Ch, found by drawing a perpendicular to CB at f meeting NM at h.

The instantaneous axis of CB is at the point O, where a perpendicular to NM through C meets AB produced.

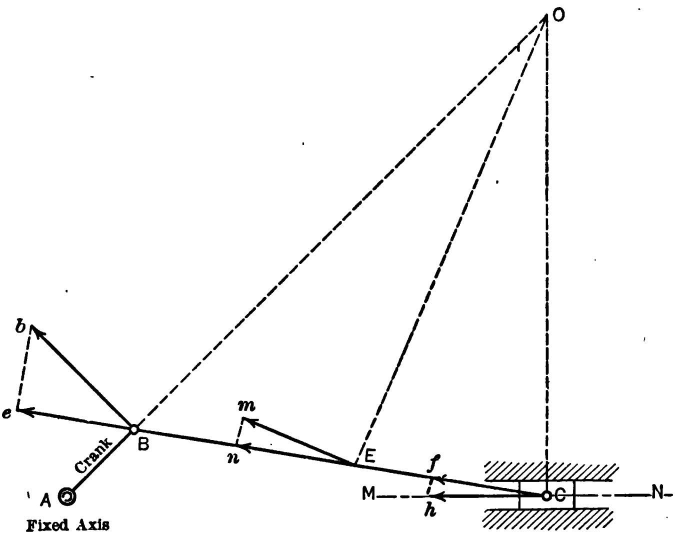


FIG. 171.

Knowing the position of this instantaneous axis the velocity of E can be found as in the preceding example.

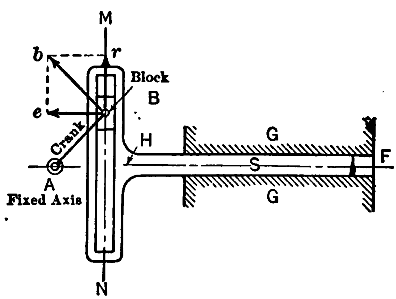


FIG. 172.

Example 53. In Fig. 172 AB is a crank turning about A. The crank pin carries a block which works in a slot whose

center line is MN. This slot is in the T-shaped head of the slider S which moves in the guides G. HF is the center line of the guides and MN is at right angles to HF. If Bb represents the velocity of B, let it be required to find the velocity with which S is moving in the guides and also the rate at which the block is slipping in the slot MN.

Solution. The component of Bb which is parallel to HF will indicate the rate at which the block is causing the slot to move, and the component of Bb which is parallel to MN will indicate the rate at which the block is slipping in the slot. Therefore Bb is resolved into the two components Be and Br (or eb) parallel to HF and MN, respectively, giving Be as the velocity of S and Br (or eb) as the rate of slipping of the block in the slot.

The correctness of the above solution can be understood if it be assumed that the point B is actually allowed to go to b and that instead of going over the path Bb it moves first to e along Be parallel to HF. In order to go there and still remain in the slot it must move the center line MN along with it until MN passes through e. Then B moves up along eb parallel to MN until it reaches b. This latter motion, since it is parallel to the slot does not move the slot at all, but is simply a motion of slipping in the slot. Of course B does not actually go to b, since it is constrained to move in a circle about A and the next instant the direction of the velocity of B will be different and the rate of slipping and the velocity of S will have changed accordingly. As has already been pointed out, however, the velocities and directions shown in the figure are the ones which obtain for the instant when the mechanism is in the position shown.

Example 54. Fig. 173 shows a mechanism exactly the same as that shown in Fig. 172, except that the center line MN is not at right angles to HF. The method of solution is the same as described in the preceding example.

Example 55. In Fig. 174, AB is a crank turning about the fixed axis A. The crank pin whose center is B carries a

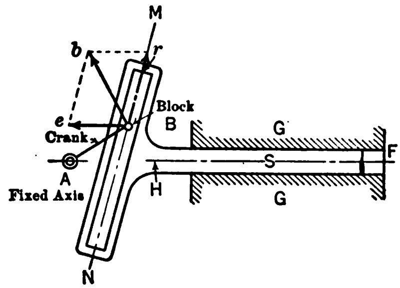


FIG. 173

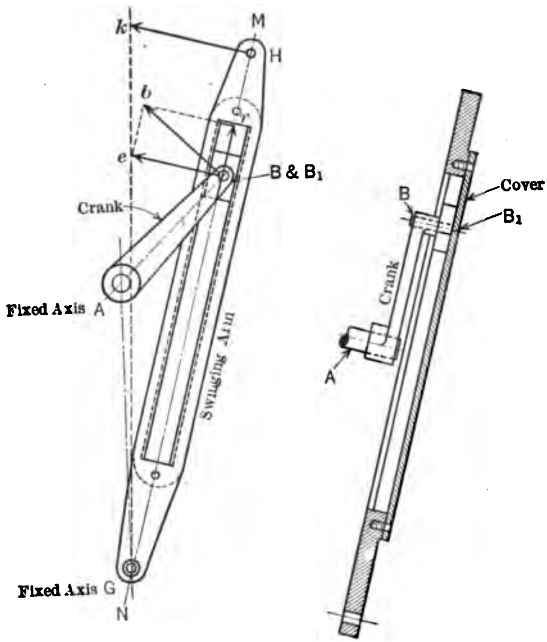


FIG. 174.

block working in a slot in the long arm which turns about the fixed axis G. The center line MN of this slot passes through G. If Bb represents the velocity of B, let it be required to find the velocity of a point H on the swinging arm, on the center line MN.

Solution. The velocity of H is, of course, the result of an angular speed of the arm caused by the motion of the block. This problem might be solved by the principle of the four-bar linkage and will be so analyzed later. At this point, however, it will be treated by means of the principles of resolution of velocities. Let it be assumed that the slot is covered on the back by a strip, as shown. Then a point B_1 on this strip which is directly in line with B is turning about the center G. It is necessary, first of all, to find the linear velocity of B_1. Bb is resclved into two components, Be and Br (or eb). Be is along the direction in which B_1 is moving, that is, perpendicular to BG. eb is parallel to the center line of the slot; the figure $Bebr$, as in the previous cases, being a parallelogram. Be then represents the velocity of B_1. The velocity of H is to Be as GH is to GB. Since GB and H are in a straight line the above proportion can be made graphically by drawing from G a straight line through e meeting at k the perpendicular to GH through H. The triangle GBe and GHK are similar, therefore HK is to Be as GH is to GB. Hence HK is the required velocity of H.

Example 56. Fig. 175 shows a mechanism similar to that in Fig. 174, except that the center line of the slot does not pass through G.

Solution. The principles involved in the solution are the same as those described for Example 55. The component Be is perpendicular to GB and not to MN. H has a velocity perpendicular to GH and bearing the same ratio to Be that GH bears to GB. The similar triangles can be constructed by laying off on GH a distance GB_2 equal to GB, drawing B_2L perpendicular to GB_2 and equal in length to Be, then drawing from G through L to meet the perpendicular

to GH through H at K. HK is then the required velocity of H.

Example 57. The mechanism in Fig. 176 is the same as that in Fig. 175, except that the slot is curved instead of

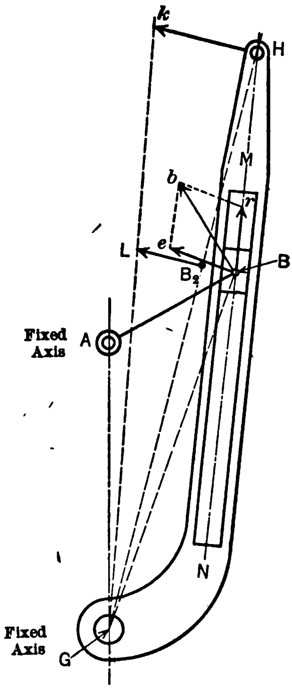


FIG. 175.

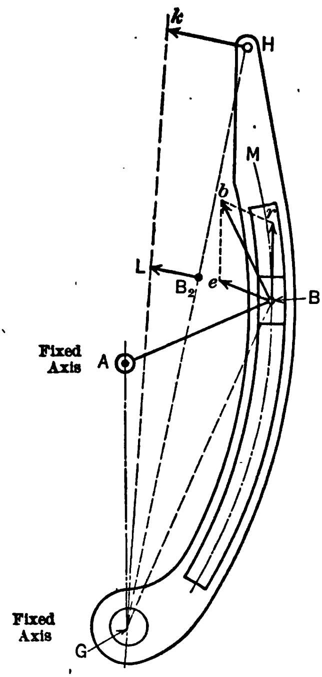


FIG. 176.

straight. The method of solution is the same. The only new point to be noticed is that the line Br is tangent to the center line of the slot at B (see § 127).

134. Parallel Crank Four-bar Linkage. In Fig. 177, the crank AB is equal in length to the crank CD and the line of centers AD is equal to the connecting rod BC. The center lines of the linkage thus form a parallelogram in every posi-

tion, provided the cranks turn in the same direction. Therefore, the perpendicular Am and Dn are always equal and the

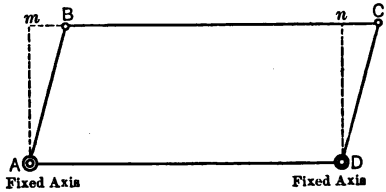


FIG. 177.

two cranks are always turning at the same angular speed (see § 131). A familiar example of this linkage is furnished by the cranks and parallel rod of a locomotive.

135. Non-Parallel Equal Crank Linkage. In the linkage shown in Fig. 178, AB is equal to CD and AD is equal to BC. Provision is made, however, to cause the cranks to turn in

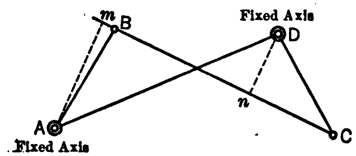


FIG. 178.

opposite directions; in which case the perpendiculars Am and Dn do not remain equal to each other. Therefore, if the crank AB turns with uniform angular speed, the crank DC has a varying angular speed, although both make one complete turn in the same length of time.

136. Centroids of Links in Non-Parallel Linkage. If in the linkage in Fig. 178, AD being the line of centers, the centroid of BC is drawn (see § 130) it will be found to be an hyperbola. Similarly, if BC is considered to be the line of centers and the centroid of AD is drawn, this will be an hyperbola like the former one.

If, on the other hand, DC is considered as the line of centers, and the centroid of AB is drawn, it will be found to be an ellipse with D and C as foci and with a major axis

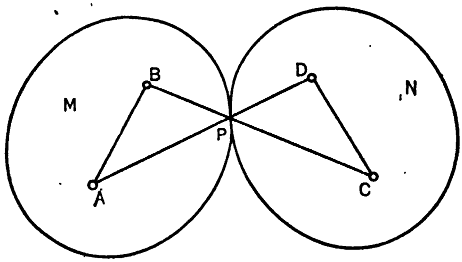


FIG. 179.

equal to AD or (BC). In a like manner, when AB is the line of centers, the centroid of CD is a like ellipse with A and B as foci. These ellipses are shown in Fig. 179.

It is possible to prove the truth of the above statement mathematically, but the proof is long, and the graphical construction is sufficient for present purposes.

Now, let it be supposed that two equal gears M and N are made whose pitch lines are the ellipses shown. Let one of these gears be pivoted at the focus A and the other gear at the focus D, P being the pitch point or point of contact of the two pitch lines. Then, if the lines AB, BC, CD, and DA are drawn on the gears and one of the gears, say M, is caused to turn, it will be found to turn the other gear and the lines AB and DC will revolve as cranks of the four-bar linkage, while the line BC will be found, at all times, to pass through the point of contact of the pitch ellipses of the gears. In other words, the four-bar linkage has been given the form of a pair of gears.

Elliptic gears designed on this basis have been used to drive the rams of machine tools, such as slotters, so as to give a slow cutting stroke to the tool, and a quicker return stroke. In applying the gears for such a purpose one of

them, as for instance, M, is on a shaft at A driven at a uniform speed from some external source of power. The other gear N is on a shaft at D to which is attached the crank or other device for moving the ram.

137. Sliding Block or Engine Linkage. In Fig. 180, if instead of using a crank at DC the end of the connecting rod carries a block pivoted to it at C which slides back and

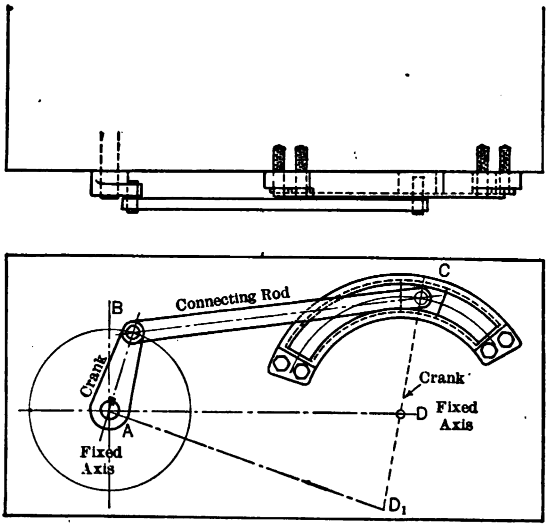


FIG. 180.

forth in the circular slot as the crank AB revolves. The center of the slot is at D, the center of the crank pin C evidently has the same motion that it would have were it guided by a crank of length DC turning about D. The mechanism, therefore, is really a four-bar linkage with the lines AB and DC as center lines of the cranks AD, as the line of centers, and BC as the center line of the connecting rod.

Let it now be supposed that the slot is made of greater radius than that shown in the figure, for example, with its

center at D_1. Then the equivalent four-bar linkage would be $ABCD_1$.

Carrying the same idea still further, let the slot be made straight. Then the equivalent center D would be at an infinite distance away. The mechanism, however, would

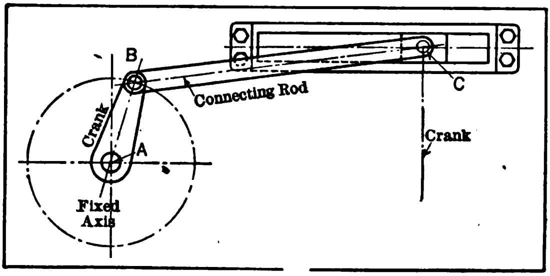


FIG. 181.

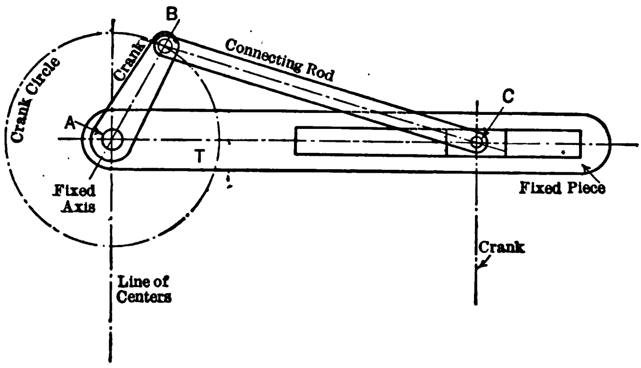


FIG. 182.

still be the equivalent of a four-bar linkage, as shown in Fig. 181, where AB is one crank, the line through C perpendicular to the slot is the other crank, BC the connecting rod, and a line through A parallel to the crank through C is the line of centers.

Fig. 182 shows the special form in which this linkage

commonly occurs, where the center line of the slot passes through the center of the shaft A. This is the mechanism formed by the crank shaft, crank, connecting rod, crosshead and crosshead guides of the reciprocating steam engine.

Linear Velocity of Crosshead. It is convenient to be able to determine the velocity of the crosshead, and hence of the piston, of a steam engine for different positions of the

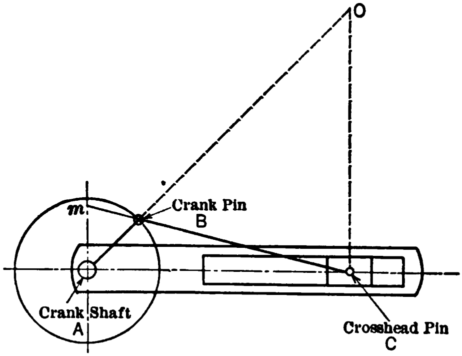


FIG. 183.

stroke when the velocity of the crank pin is known. This may be done by means of a mathematical equation. For practical purposes, however, a graphical construction is sufficiently accurate. In Fig. 182

$$\frac{\text{Linear velocity of } C}{\text{Linear velocity of } B}=\frac{OC}{OB}. \quad . \quad . \quad . \quad (65)$$

Through A draw a line perpendicular to the center line AC, and extend the center line of the connecting rod to cut this line at m. Then, the triangles OCB and mBA are similar. Hence, $\frac{OC}{OB}=\frac{Am}{AB}$.

Substituting this in Eq. (76) gives

$$\frac{\text{Linear velocity of } C}{\text{Linear velocity of } B}=\frac{Am}{AB}; \quad . \quad . \quad . \quad . \quad (66)$$

or in words, *The linear velocity of the crosshead or piston of a steam engine is to the linear velocity of the crank pin as the distance between the crank shaft and the point where the connecting rod cuts the perpendicular through the center of the crank shaft is to the length of the crank.* . . . . . (67)

Displacement of Crosshead. To find the distance which the crosshead is from the end of its stroke for any known position of the crank the following simple construction is used. In Fig. 184, A is the center of the crank shaft, AB

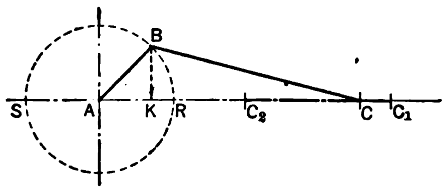


FIG. 184.

the crank. Let it be required to find the displacement, from the end of the stroke, of the crosshead when the crank is in the position shown. Draw a circle with A as a center and a radius AB. This circle is the path of the center of the crank pin. From the points R and S where the crank-pin circle cuts the main center line, with a radius equal to the length of the connecting rod, cut the center line at C_1 and C_2. These points are the two extreme positions of the center of the crosshead pin. From B with a radius equal to the length of the connecting rod cut the center line at C. Then C is the position of the center of the crosshead pin when the crank pin is at B, and if the crank is turning left-handed the crosshead has moved the distance C_1C from its extreme position.

If the motion of the crosshead were harmonic (see § 11), its displacement would be equal to RK, where K is the foot of a perpendicular let fall from B to the center line.

It is apparent that C_1C is not equal to RK. In other words, with the crosshead connected to the crank pin by a connecting rod of ordinary length, the crosshead does not move with harmonic motion if the crank pin turns at uniform speed. It will be found, however, that the longer the connecting rod for a given length of crank, the less the motion of the crosshead varies from harmonic motion.

If a device similar to that shown in Fig. 185 is used, in which a rigid rod is attached to the crosshead, this rod having

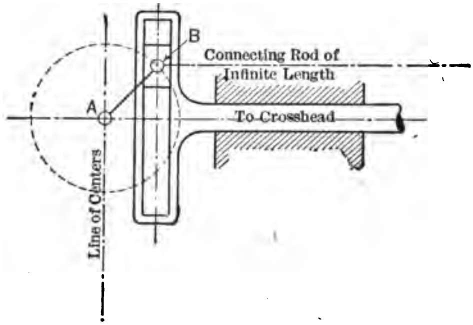


FIG. 185.

a slot in it at right angles to the line of motion of the crosshead and embracing a block pivoted on the crank pin, then the crosshead would have harmonic motion if the crank turned at uniform speed.

This device is the equivalent of a connecting rod of infinite length, and the whole mechanism may be thought of as a four-bar linkage in which only one of the links, namely, the crank AB, is a finite quantity. The line of centers and the infinite connecting rod are indicated in the figure. The other crank is an imaginary line parallel to the line of centers at an infinite distance away.

138. Swinging or Rocking Block Four-bar Linkage. Referring to Fig. 182, if, instead of considering the piece T as the stationary piece, giving the four-bar linkage as there

indicated, the piece BC is made stationary, the linkage becomes that shown in Fig. 186. Here BA is a crank but B is now the crank pin. The pin C is a fixed axis with the

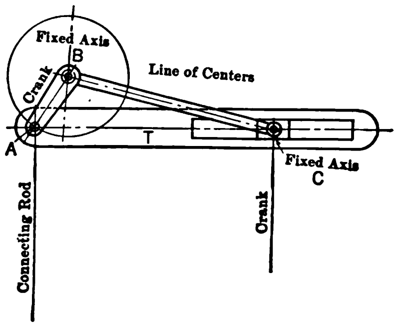


FIG. 186.

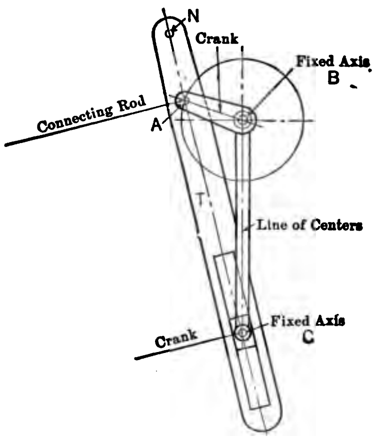


FIG. 187.

block swinging on it. As the crank BA revolves the piece T oscillates and, at the same time, slides back and forth over the block.

This mechanism is applied in a modified form as a quick return motion in certain machine tools, particularly shapers.

Fig. 187 is the same as Fig. 186, drawn with the line BC vertical and with the piece T extended with a connection at N to drive the ram of the shaper. The apparent objection to using the mechanism in this form is that the point N

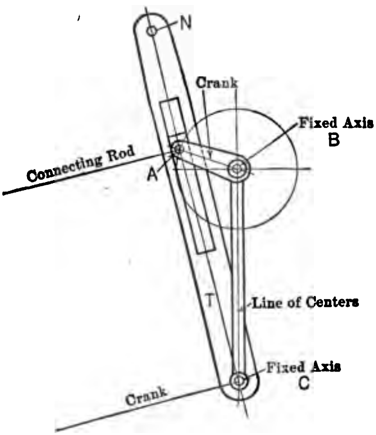


FIG. 188.

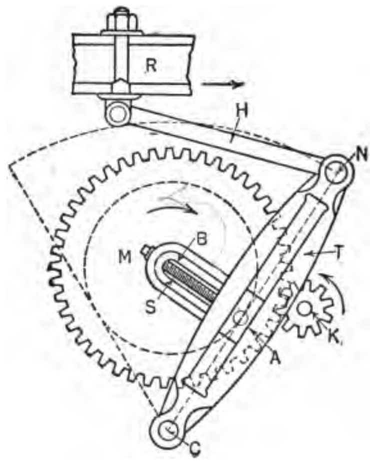


FIG. 189.

moves up and down at the same time that it swings. Now, the only real purpose of the sliding of the piece T over the block is to allow the distance between A and C to change as the crank BA revolves. This is accomplished equally well if the block is pivoted on the pin A and the arm T swings on a pin at C, as shown in Fig. 188. Fig. 189 shows one way in which the mechanism is actually used. The crank BA is in this case the gear M. A being a pin fast to M and carrying a block which works in a slot in T. M is driven by the pinion K. The link H connects N to the ram R which carries the cutting tool. The length of the crank BA may be changed by turning the screw S, thus changing the length of stroke of the tool.

139. Whitworth Quick-return Mechanism. If BA (Fig. 182) is made the fixed link, the four-bar linkage becomes that

shown in Fig. 190. It will be noticed that this is similar in appearance to the swinging block mechanism shown in Figs. 187 and 188, except that in this case the driving crank

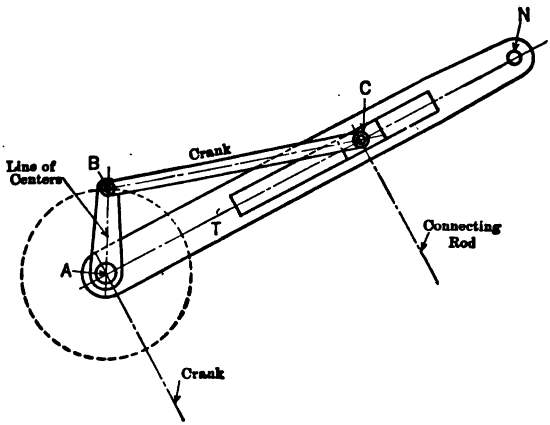


FIG. 190.

is longer than the line of centers, so that the piece T makes a complete revolution when BC turns once, whereas the arm T in Figs. 187 and 188 merely oscillates. The linkage in Fig. 190 is also used as a device for driving the ram of a shaper and is given the name *Whitworth Quick-return Mechanism.*

140. Pantograph. Fig. 191 shows a device known as a *pantograph.* It is essentially a four-bar linkage with only one point fixed. AD is equal to BC and AB is equal to DC. The points A, E, and F are on a straight line, F being on a prolongation of DC. If F is moved to F_1 along the straight line FF_1, E will move to E_1 along the line EE_1. The length of EE_1 is to the length of FF_1 as AE is to AF. Similarly, if F is moved to F_1 along the curved path, E will follow a path which is an exact duplicate of the path of F, except that it is reduced in the ratio of AE to AF. Fig. 192 shows another pantograph in which K is the fixed axis and H, E and F

are moving points, all on a straight line passing through *K*. If any one of these points is moved over a certain path, each

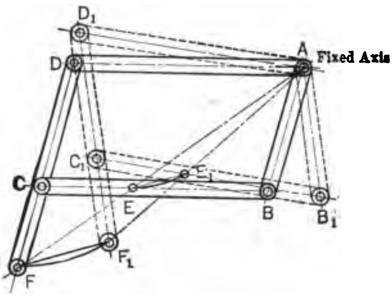


FIG. 191.

of the others will move over a similar path, increased or reduced in the ratio of their respective distances from the

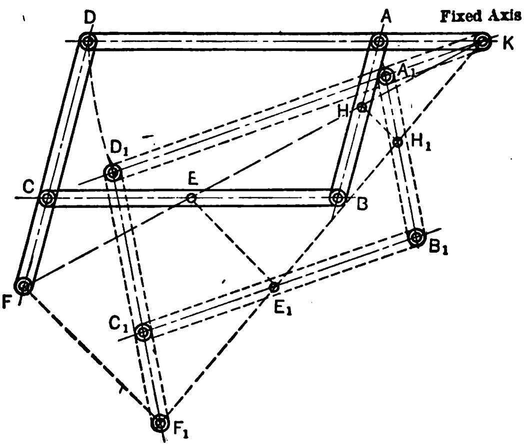


FIG. 192.

center. The pantograph is used for copying maps and other drawings at reduced or enlarged scales.

APPENDIX

PROBLEMS FOR SOLUTION

Abbreviations:—r.p.m. = revolutions per minute. r.p.s. = revolutions per second. l.v. = linear velocity or linear speed. a.v. = angular speed. dia. = diameter. ft.p.m. = feet per minute. ft.p.s. = feet per second. R.H. = right handed. L.H. = left handed.

1. The flywheel of a steam engine is 6 ft. in diameter and turns at the rate of 300 r.p.m.

(*a*) What is the l.v. of the surface of the wheel in feet per minute?

(*b*) What is the a.v. in radians per minute?

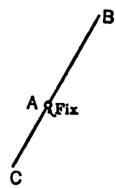
(*c*) How many inches from the center is a point which has a l.v. of 31,416 inches per minute?

2. A pulley 30 in. dia. has an a.v. of 40 radians per second. How many r.p.m. does it make and what is the l.v. of its outer surface in ft. p.m.? Another pulley on the same shaft has a surface speed of 50 ft. p.s. What is its diameter?

3. An emery wheel 12 in. dia. has a surface speed of 5500 ft. p.m. How many r.p.m. does it make? What is its a.v. in radians per second?

4. The l.v. of two points on a disc revolving at the rate of 330 r.p.m. are 600 inches per second and 2000 ft. p.m. respectively. How far are these points from the axis?

5. $AB = 12$ in., $AC = 8$ in. If the bar BC turns about A at the rate of 12 r.p.s. what is its a.v. in radians per second? What is the l.v. of B and of C in inches per second?



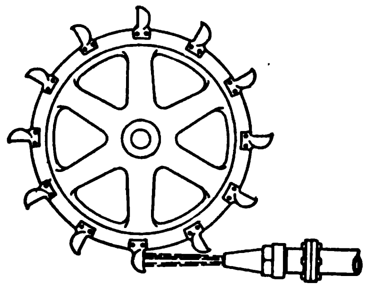
Prob. 5.

6. The shaft of a milk separator has an angular velocity of 6283.2 radians per minute. How many r.p.m. does the shaft make? How many inches from the center is a point on the bowl having a l. v. of 3927 ft. p.m.?

7. In turning steel shafting a cutting speed of 80 ft. p.m. is allowable. How fast must a piece of 6-in. shaft turn to give this speed?

8. The Pelton water wheel shown in the figure is driven by a jet of water having a velocity of 90 feet per second. The l.v. of the

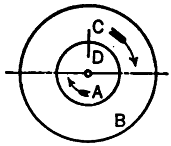
center line of the buckets is 0.6 that of the water. What is the diameter in inches to the center line of the buckets if the wheel is fast to a shaft that turns 100 r.p.m.?



Prob. 8.

9. The difference between the linear velocities of two points on a radial line drawn on the side of a revolving disc that turns 100 r.p.m. is 2513.28 in. per minute. Find the distance between the points.

10. Two discs A and B are rotating on the same axis in the direction indicated by arrows, but at different speeds. A has an a.v. of 75 radians per minute, while B make 12.74 r.p.m. Calculate the number of degrees that the reference marks C and D will be apart after B has made 30 complete revolutions. Make a sketch showing their relative position at that time.

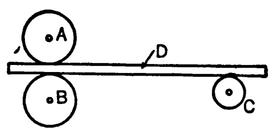


Prob. 10.

11. Two axes A and B are 8 in. apart. A is to make 10 revolutions R.H. while B makes 30 revolutions L.H. Find radii of rolling cylinders to connect them. (Calculate and solve graphically.)

12. Two shafts A and B are to be connected by cylinders in pure rolling contact and to turn in the same direction. If the turns of B are to the turns of A as 3 to 8, and the axes are 4 in. apart, find the *diameters* of the two cylinders. Solve both graphically and by calculation.

13. A and B are the feed rolls of a wood planer and C is the revolving cutter, 4 in. dia. D is a board being fed to the right by the feed rolls at the rate of 20 ft. p.m. C turns L.H. at the rate of 2000 r.p.m. What is the cutting speed? What would the cutting speed be if the cutter turned R.H.?



Prob. 13.

14. Two shafts, $5\frac{1}{2}$ in. apart, are to be connected by a pair of cylinders in rolling contact, so that the shafts will turn in the same direction. A point in the surface of the smaller cylinder has a l.v. of 5024 in. per minute, and the larger cylinder has an a.v. of 628 radians per minute. What are the diameters of the cylinders and how many r.p.m. does the smaller cylinder make?

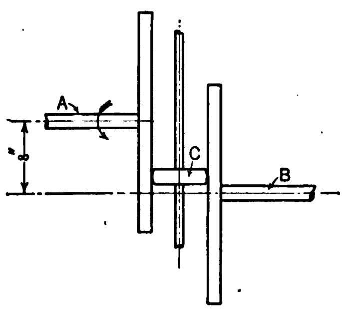
15. The shafts A and B are 10 in. apart and are to be connected by two rolling cylinders. A turns 150 r.p.m. and B turns 350 r.p.m., both cylinders turning as shown. Find diameters for A and B. Then, if diameter of B remains unchanged, and A is to turn 150 r.p.m. in the opposite direction to that shown, what is the new diameter of A and how are the r.p.m. of B affected? Calculate and solve graphically.

Prob. 15.

16. Two shafts A and B meeting at an angle of 30° are to be connected by a pair of cones so that B is to make 80 turns while A makes 30 turns. Find graphically the element of contact when the shafts turn in opposite directions and when they turn in the same direction. If the diameters of the base of the smaller cone is $1\frac{1}{4}$ in. in each case calculate the diameter of the base of the larger cones and then draw the cones full size.

17. Two shafts A and B meeting at an angle of 105° are to be connected by a pair of cones in external contact so that the a.v. of A is three-fourths the a.v. of B. Find graphically the element of contact and indicate the relative direction of rotation of the two shafts. Assume a base diameter of 4 in. for the smaller cone, calculate the diameter of the base of the larger, and draw the cones to scale.

18. Referring to Fig. 24, page 28 of the text, disc F is 3 in. diameter and shaft T turns 600 r.p.m. How large must disc H be made to give a range of speed of shaft S, attached to H of 200 to 2000 r.p.m.? How far in on H must F move to give this range? How far must F be moved to the left of the center of H to give a speed of 1300 r.p.m. in the opposite direction?

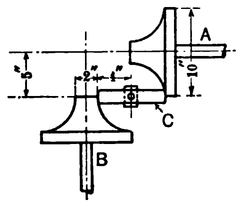


Prob. 19.

19. A and B are two parallel shafts in the same vertical plane, carrying on their ends two cylindrical discs 2 ft. diameter. The disc

C is 6 in. diameter and is on a vertical shaft in the same plane as A and B. C may be held in different positions on its shaft. A turns at a constant speed of 90 r.p.m. as shown. How many r.p.m. does B make and in what direction when C is (*a*) 4 in. above shaft A, (*b*) 4 in. below shaft A, (*c*) 6 in. below shaft A?

20. A and B are two shafts at right angles, in the same vertical plane. C is a disc carried by a supporting yoke on a horizontal shaft arranged, so that C is always in contact with the equal conoids on A and B. A turns at a constant speed of 60 r.p.m. What is the maximum speed of B? What is the minimum speed of B? What is the speed of B when the yoke supporting C has turned 30° from its present position?



Prob. 20.

21. Involute gears 2 Pitch $22\frac{1}{2}$° obliquity. A 14-tooth pinion turning clockwise is to drive a 20-tooth pinion. Addendum is $\frac{7}{16}$ in., clearance is $\frac{1}{16}$ in., width of tooth is equal to width of space, no backlash. Draw the pitch, base, root and addendum circles and show the path of contact and the angles of approach, recess and action. Could the arc of approach equal three-fourths the circular pitch, and could the arc of recess equal the circular pitch, disregarding the given addendum? Why? Show clearly. Draw one complete tooth on each.

22. Involute gears 3 pitch, 15° obliquity. A pinion with 18 teeth is to drive a rack. The addendum on the rack is to be as much as the pinion will allow. The addendum on the pinion is to give an arc of recess of $1\frac{1}{4}$ times the circular pitch Draw the pitch, base, root and addendum circles of the pinion and the addendum and root lines of the rack, and show the path of contact if the pinion drives left handed. The 18-tooth pinion is also to drive an annular of 36 teeth, the addendum on the annular to be as much as the pinion will allow. Draw the pitch, base, root, and addendum circles for the annular on the side of the 18-tooth pinion opposite to the rack. All clearances are $\frac{1}{24}$″.

23. (*a*) An involute gear of 15° obliquity, 30 teeth, 2 pitch, is to have arcs of approach and recess each equal to the circular pitch. Clearance $\frac{1}{16}$ in. Draw two complete teeth. Indicate angles of approach and recess.

(*b*) Find the smallest possible gear to run with the above gear, giving the number of its teeth. Indicate the path of contact if the 30-tooth gear drives and turns clockwise.

24. Involute gears 2 pitch, 15° obliquity. A pinion with 10 teeth

is to drive another pinion and provide an arc of action $1\frac{1}{4}$ times the circular pitch. Find the number of teeth on the smallest pinion it can so drive and then see if the required arc of recess is possible.

25. Involute gears, $\frac{3}{4}$ pitch, 20° obliquity. A 15-tooth gear turning clockwise is driving a rack. There is to be no backlash and teeth are the same width as the spaces, clearance is $\frac{1}{16}$ in. The addendum of the rack is to be as much as the gear will allow, while the addendum of the gear is such as to give a total arc of action of $1\frac{1}{2}$ times the circular pitch. Draw the pitch, base, root and addendum circles, and show the path of contact; also show the angles of approach and recess for the gears. Draw two teeth on both gear and rack.

26. A pair of speed cones carrying a crossed belt is used to connect two shafts 25 in. between centers. The driving shaft has a constant speed of 150 r.p.m. Driven shaft has speeds of 100, 150, 200 and 300 r.p.m. as the belt is moved along the cones equal distances. Smallest diameter of driving cone is to be 10 in. Find all diameters for cones and plot cones $\frac{1}{4}$ size if their length is 20 in.

27. Two parallel shafts are connected by a pair of speed cones carrying a crossed belt. The driving shaft has a constant speed of 135 r.p.m. The driven shaft is to have a range of speeds from 45 r.p.m. to 300 r.p.m., the speeds to increase in arithmetical progression as the belt is moved equal distances along the cones. The smallest diameter of *either* cone is 3 in. Find the diameters of the cones at the ends and at two intermediate points. Plot cones $\frac{1}{4}$ size if their length is 24 in.

28. A drill carrying a 3-step pulley is driven by an open belt from a pulley of the same size on a countershaft 4 ft. below the centerline of the drill pulley. The maximum and minimum speeds of the drill are to be 413 and 75 r.p.m. Find proper speed for counter shaft and diameters of all steps on the pulleys. Diameter of smallest step is to be 6 in.

29. A lathe having a five step pulley is driven by an open belt from a pulley of the same size on the countershaft, 3 ft. above. The countershaft is to have a constant speed, and the lathe is to have speeds of 60 r.p.m. and 135 r.p.m. when the belt is on the steps either side of the center step. If the minimum speed is 40 r.p.m. and the smallest diameter 4 in., find proper speed of countershaft, maximum speed of lathe, and diameter of all steps on the pulleys.

30. A 3-in. belt is designed to stand a difference in tension, between the tight and slack sides, of 50 lb. per inch of width. Find the least speed at which it can be driven to transmit 20 horse-power.

31. A belt 2 in. wide connects two pulleys, one of which is 6 in. diameter, and turns 600 r.p.m. If the belt transmits $3\frac{1}{2}$ H.P. and if

the tension on the tight side is $2\frac{1}{2}$ times that on the loose side, find the tension per inch of width, with which the belt was put on the pulleys, assuming the sum of the tensions to remain constant.

32. Two equal three step pulleys are 50 in. apart. The largest diameter on each is 10 in. and the driven shaft is to have a maximum speed of 100 r.p.m. and a minimum speed of 49 r.p.m. Find (*a*) the speed of the driving shaft if it is constant and find all diameters of other steps if an open belt is used. (*b*) If the belt is transmitting $1\frac{1}{2}$ H.P. and is on the 10-in. step of driving pulley, what is the effective pull? (*c*) If a belt 4 in. wide is used and the maximum stress on the belt per inch of width is 100 lb., find with what initial tension per inch of width the belt was put on. Assume T_1+T_2 constant.

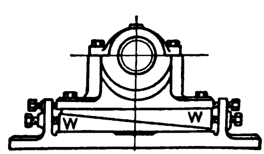
33. What is the width of a double belt transmitting 45 H.P. when running on a 30 in. pulley making 550 r.p.m.? The belt is put on with an initial tension of 70 lb. per inch of width and the working stress is not to exceed 105 lb. per inch of width. Assume T_1+T_2 constant. Also calculate the width of belt using the millwright's rule.

34. A rope drive composed of 18 ropes, is to transmit 120 H.P. If the pitch diameter of one of the pulleys is 48 in., and if it has an a.v. of 1500 radians per minute, what is the effective pull in each rope?

35. A water wheel developing 500 H.P. drives the main shaft of a mill through a rope drive. The sheave on the water wheel shaft is 48 in. diameter and the speed of the main shaft is 200 r.p.m. Design a rope drive, using tables in text book to fulfil the above conditions, also give the width of sheave. Compare this width with that of the face of a pulley to transmit the same power, using the millwright's rule for a double leather belt.

36. A motor running 600 r.p.m. transmits 4 H.P. through a chain drive. The effective radius of the driving sprocket is 3 in. What is the effective pull in the chain?

37. In the adjustable pillow block shown in the figure, the bearing is raised or lowered by means of the wedges W. The wedges have a taper of $\frac{1}{2}$ in. per foot and the lead of the screws is $\frac{1}{12}$ in. How many times must each of the screws be turned to raise the center line of the shaft $\frac{1}{4}$ in., assuming that both wedges move the same amount?



Prob. 37.

38. In Fig. 114, page 126, nut N is free to move horizontally between the guides GG. Lead of screw is $\frac{1}{4}$ in. R.H. How many turns of the handle K are necessary to move N

to the right $3\frac{3}{8}$ in.? Must the handle be turned R.H. or L.H. as seen from the left? How far will N move for 26 turns of the handle K and which way if handle is turned R.H. as seen from the left?

39. In the jack screw shown in Fig. 115, page 126, find W if P is $\frac{1}{4}$ in. R. H.; $R=3$ ft. 6 in. and $F=50$ lb. Neglect friction. (Take $\pi=\frac{22}{7}$).

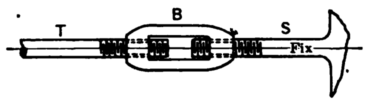
40. In the jack screw Fig. 115, page 126, find the lead of the screw if the pull is 70 lb.; $R=4$ ft., and $W=19{,}008$ lb. Allow for a friction loss of 70 per cent. (Take $\pi=\frac{22}{7}$).

41. A building is to be raised by four jack screws, Fig. 115, page 126, one at each corner. The lead of one is $\frac{1}{4}$ in., of another $\frac{1}{3}$ in., while the third and fourth require one complete turn to raise the jack $\frac{1}{2}$ in. and $\frac{3}{8}$ in. respectively. For one turn of the lever of the first screw how many turns must the others make to raise the building uniformly?

42. In the mechanism shown in Fig. 118, page 129, if $P_1=\frac{3}{16}$ in., L.H., $P_2=\frac{1}{8}$ in. L.H., how many turns of A are necessary and in which direction, to lower the slide $\frac{1}{2}$ in.? If a push and a pull each 28 lb., be applied at opposite sides of the hand wheel A what is the pressure on W if 75 per cent. is lost in friction?

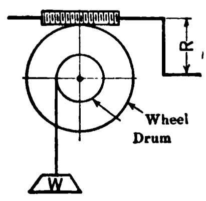
43. Referring to Fig. 117, page 128, what load W, suspended from the nut N, can be raised by a force of 60 lb. applied at F? Screw is double-threaded and has a lead of $\frac{1}{2}$ in. Assume friction loss of 40 per cent. R is 21 in.

44. Referring to Fig. 116, page 127, 13 turns of K are to move N 3 in. to the right; $P=0.13$ in. R.H. Find P_1. Is P_1 R.H. or L.H.? Must K turn R.H. or L.H. as seen from the left?



Prob. 45.

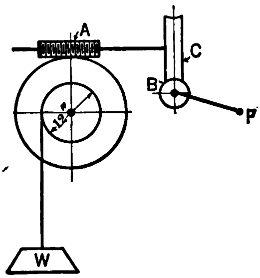
45. Screw S has 9 threads per inch and T has 10 threads per inch, both R. H. single threads. A force of 50 lb. acting at the end of a wrench 14 in. long, which is applied to the turnbuckle B, produces a tension of 15,840 lb. in T. Find the percentage friction loss. (Take $\pi=\frac{22}{7}$).



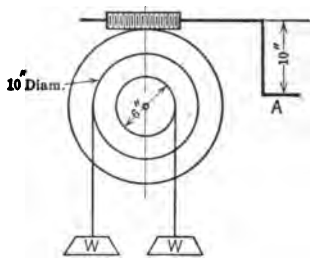
Prob. 46.

46. In a triple-threaded worm and wheel, let the diameter of the drum be 14 in. How many teeth must the wheel have if 30 turns of the worm are to move W 20 in.? If $R=16\frac{2}{3}$ in., what force must be applied to the crank if W equals 8000 lb. actually lifted, 60 per cent. being the loss due to friction? (Take $\pi=\frac{22}{7}$).

47. Worm A is double-threaded and its wheel has 36 teeth. Worm B has a pitch of $\frac{5}{8}$ in. The force F, at the end of a 16-in. handle on B, is 20 lb. and W is 25,344 lb. If 40 per cent. is lost in friction, what is the diameter of the worm wheel C? (Take $\pi = \frac{22}{7}$).



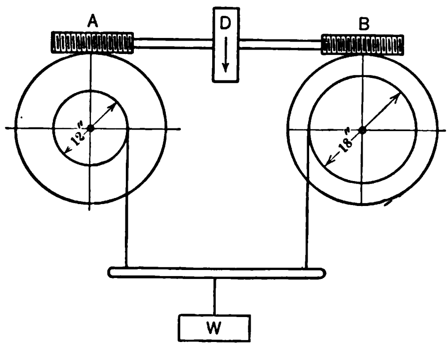
Prob. 47.



Prob. 48.

48. A linear motion of the crank A of 6283.2 in. produces a difference of level of 125.664 in. in the weights W. If the worm is double-threaded, how many teeth are there on the worm wheel?

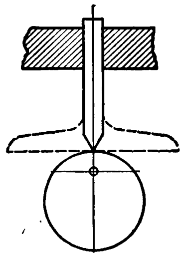
49. D is a pulley 48 in. in diameter fast to a shaft driving worms A and B. Worm A is triple-threaded, and worm B has a pitch of $1\frac{1}{2}$ in. Is A R.H. or L.H. and is B R.H. or L.H., if W is to rise when



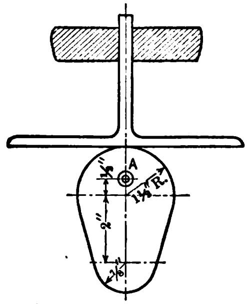
Prob. 49.

pulley turns as shown? The belt on D has a linear velocity of 600 ft. p.m. If the two strands of rope rise uniformly and W rises 3 ft. p.m., find the number of teeth on the worm wheel driven by worm A, and diameter of worm wheel driven by B. (Take $\pi = 3$).

50. This plate cam is a circular disc 4 in. in diameter, turning uniformly about an axis 1¼ in. from the center of the disc. The follower is pointed, as shown in full lines. Plot a curve whose ordinates represent displacements of the follower (full size) and whose abscissæ represent angular motion of the disc (⅜ in. = 15°). Plot points every 15° for a complete turn. Plot on the same diagram a similar curve, using the follower shown in dotted lines, instead of the pointed one.



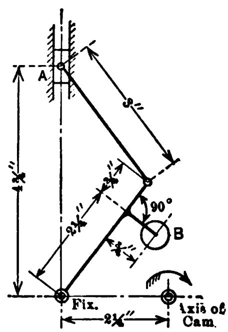
Prob. 50.



Prob. 51.

51. This plate cam is made up of arcs of two circles and their common tangents and turns about the fixed axis *A*. Plot a curve whose ordinates represent the motion of the cam. Plot points every 15° for one-half turn of the cam. Ordinates are to be full size and abscissæ are to be ⅜ in. = 15°. Plot on the same diagram a curve showing the displacements of the follower, if its motion were harmonic.

52. Draw the pitch line of a plate cam to give motion to a point in a vertical line passing through the center of the cam as follows: Still during ⅓ turn of cam, down uniformly 2 in. during ⅓ turn, and up 2 in. harmonic during ⅓ turn. Highest position of follower 3 in. above axis of cam. Cam turns uniformly R.H.



Prob. 53.

53. Find the outline of a plate cam turning uniformly right-handed to give block *A* the following motion: Rise uniformly 1 in. in ⅓ rev.; still ⅓ rev., drop 1⅜ in. at once, rise ⅜ in. uniformly in ⅓ rev.; cam is to drive roller *B* ½ in. in diameter.

54. $AC = 4\frac{3}{4}$ in., $AB = 2\frac{1}{2}$ in., $BD = 1$ in., $AO = 4$ in., Cam acts on lever AC at the point D. If slide S is to have the following motion find the outline of the cam: Up $1\frac{1}{4}$ in. with uniform motion in $\frac{1}{3}$ turn of the cam, still $\frac{1}{6}$ turn, drop at once to original position, still $\frac{1}{2}$ turn.

55. Draw the pitch line of a plate cam which, when acting on a roller $\frac{3}{4}$ in diameter pivoted at C, shall give D the following motions: Up $1\frac{1}{4}$ in. in $\frac{1}{3}$ of a turn with uniform motion, down $1\frac{1}{4}$ in. at once, up $1\frac{1}{4}$ in. with harmonic motion in $\frac{1}{3}$ of a turn and down $1\frac{1}{4}$ in. with "gravity" motion in $\frac{1}{3}$ of a turn. Cam turns uniformly as shown. D is in its lowest position when AB is horizontal and path of D is vertically above the axis.

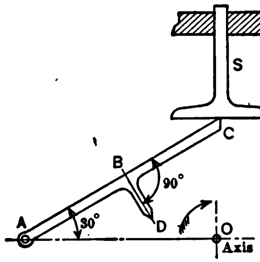
56. Design a positive motion plate cam to drive the slide S, which can move only in a horizontal direction as follows: 1 in. to the left with uniform motion in $\frac{1}{3}$ of a turn of the cam, back to the initial position with harmonic motion in $\frac{1}{6}$ of a turn, $\frac{3}{4}$ in. to right with "gravity" motion in $\frac{1}{4}$ turn, and back to the initial position with "gravity" motion in $\frac{1}{4}$ turn. Groove in side of cam to fit a roller $\frac{5}{8}$ in. in diameter, pivoted on the pin C. Cam turns uniformly L.H.

57. A and B are two rollers $\frac{3}{4}$ in. diameter attached to the same frame. The rollers are in the same plane and both are to be in contact with a single plate cam. Find the outline of the cam if the frame is to be raised 1 in. with harmonic motion in $\frac{1}{2}$ revolution of the cam.

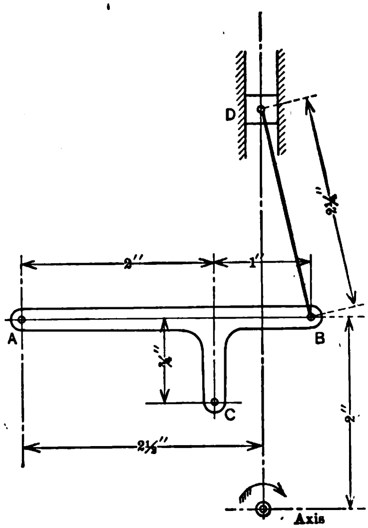
58. Draw the projection of the pitch line of a cylindrical cam 4 in. long and $2\frac{1}{2}$ in. diameter, which shall give the following motion to a point in a line parallel to and above the axis of the cam. Starting $\frac{3}{4}$ in. from the left-hand end of the cylinder, the follower is to move $2\frac{1}{2}$ in. to the right with uniform motion in $\frac{1}{3}$ of a turn, to remain still $\frac{1}{6}$ of a turn, and to return to initial position with uniform motion in $\frac{1}{2}$ a turn. Cam turns uniformly right-handed as seen from the right.

59. A cylindrical cam, 3 in. outside diameter, is to move a roller which is above and moves parallel to the axis of the cam, as follows: To the right 2 in. with "gravity" motion in $\frac{1}{3}$ turn of the cam, still $\frac{1}{6}$ turn, to the left 2 in. with "gravity" motion in $\frac{1}{3}$ turn, still $\frac{1}{6}$ turn. The roller is to be a cylinder $\frac{3}{4}$ in. diameter. Groove in cam is to be $\frac{3}{4}$ in. deep. Draw the development of that part of the cam during which motion takes place, for both outside and bottom of the groove.

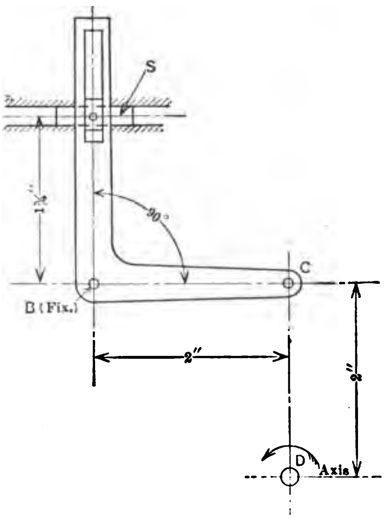
60. Draw the projection of the pitch line of a cylindrical cam 4 in. long and $2\frac{1}{2}$ in. diameter which shall give the following motion to a point in a line parallel to and above the axis of the cam. Starting $\frac{3}{4}$ in. from the left-hand end of the cylinder the follower is to move $2\frac{1}{2}$ in. to the right with harmonic motion in $\frac{1}{3}$ of a turn of the cam, to remain still $\frac{1}{6}$ of a turn, and to return to initial position with "gravity"



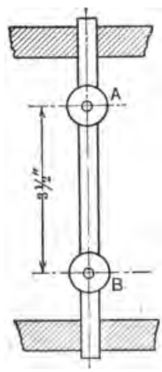
Prob. 54.



Prob. 55



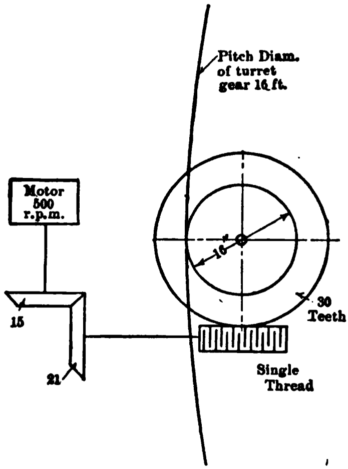
Prob. 56.



Prob. 57.

motion in ½ a turn. Cam turns uniformly right-handed as seen from the right.

61. A cylindrical cam 3 in. outside diameter, turning R.H. as seen from the right, is to move a roller, which is above and moves parallel to the axis of the cam, as follows: To the right 1¼ in. with uniform motion in ⅜ turn of the cam, still ⅛ turn, to the left 1¼ in. with uniform motion in ⅜ turn, still ⅛ turn. The roller is a cylinder ¾ in. diameter. Groove in cam is ⅜ in. deep. Draw the development of the part of the cam during which the motion takes place, for both outside and bottom of the groove. Also draw elevation of the cam, showing groove for first half turn.



Prob. 64.

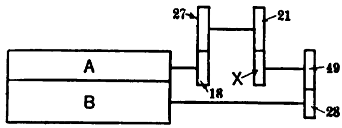
62. A wheel of 96 teeth drives another of 15 teeth which is on a shaft A turning 60 r.p.m. This shaft A drives another shaft B through gears, so that B makes a revolution in $\frac{1}{5}$ of a second. Shaft B drives a shaft C by means of a belt, the diameter of the driving pulley being 4 in., while that of the driven pulley is 10 in. How many revolutions does the first wheel make while the 10 in. pulley makes 7 revolutions?

63. In a crane the chain barrel is driven by a motor on the spindle of which is keyed a pinion of 14 teeth. This gears with a wheel of 68 teeth keyed to the same spindle as a pinion of 12 teeth. The 12-tooth pinion gears with a wheel of 50 teeth keyed to the same spindle as a wheel of 25 teeth, and the latter gears with a wheel of 54 teeth

keyed to the chain barrel spindle. Chain barrel is 16½ in. diameter. Sketch the arrangement, and find r.p.m. of motor when 20 ft. of chain are wound on the drum per minute.

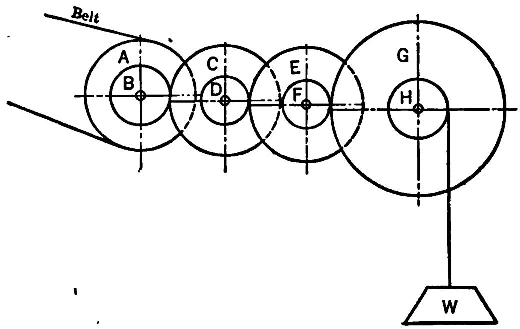
64. Find time for ½ revolution of turret.

65. *A* and *B* are two cylinders. Diameter of $A = 5\frac{1}{2}$ in. Diameter of $B = 10$ in. Find number of teeth in gear *X* if the surface speed of *A* is 110 per cent. that of *B*.

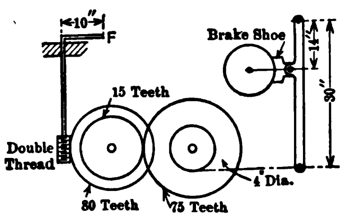


Prob. 65.

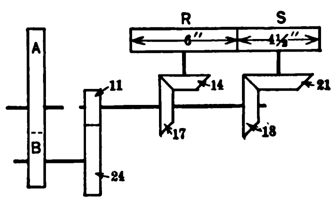
66. The effective pull in the belt is 250 lb. Weight $W = 7000$ lb. *A* is 32 in. in diameter while *H* is 20 in. in diameter. *B*, *D*, and *F* each have 18 teeth. *E* has 63 teeth; *G* has 50 teeth. If there is a friction loss of 60 per cent., how many teeth must there be in the gear *C*?



Prob. 66.



Prob. 67.



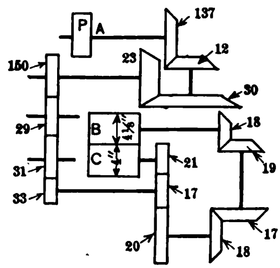
Prob. 68.

67. Force $F = 42$ lb. Find pressure at brake shoe if 80 per cent. is lost in friction.

68. *A* is an annular gear having 77 teeth, driving pinion *B* having

12 teeth. Numbers of teeth on the other gears as are given in the figure. If A makes 15 r.p.m. find the rate of slip between the cylinders R and S in feet per second.

69. Shaft A is driven by a belt on the pulley P which is 12 in. diameter. If the belt has a speed of 2500 ft. p.m., what will be the rate of slipping between the surfaces of B and C in ft. p.m.?

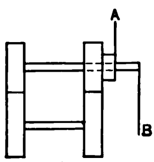


Prob. 69.

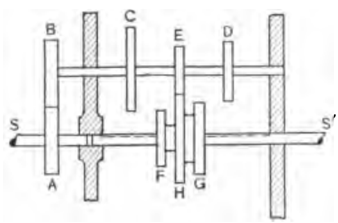
70. If a lathe is driven by a four-step cone pulley, design a set of back gears for it so that the speeds shall be in geometrical progression. Range of speeds 4 r.p.m. to 512 r.p.m. Do not use any gears of less than 20 teeth. Make a sketch showing the arrangement.

71. $\frac{\text{Rev. } B}{\text{Rev. } A} = \frac{19}{1}$; find suitable numbers of teeth for the four gears of this train, having all of the same pitch. No gear to have more than 75 teeth nor less than 10 teeth.

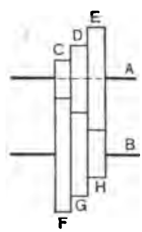
72. Shaft S has a constant speed of 100 r.p.m. Gears F, G, and H form a unit free to slide, but not to turn on shaft S_1. S_1 is to have speeds of 20, 200 and 860 r.p.m. Gear H has 80 teeth. Find numbers of teeth on all gears if they are all of the same pitch, and if gears A and B are equal. The slowest speed of S_1 is when E and H are in mesh.



Prob. 71.



Prob. 72.



Prob. 73.

73. The sketch shows a nest of spur gears, each pair being always in mesh and all gears of the same pitch. F, G, and H form a unit keyed to shaft B. C, D, and E are loose on shaft A, but may be locked to the shaft one at a time. Find the numbers of teeth on all gears if the revolutions of B for one of A are to be $\frac{1}{2}$, $1\frac{1}{4}$, and $2\frac{1}{2}$ as C, D,

and E are keyed to A respectively. No gear to have less than 14 teeth.

74. In a screw cutting train, Fig. 147, page 165, find the proper numbers of teeth for the change gears C and D to cut threads of from 4 to 12 per inch both inclusive. Lead screw has a lead of $\frac{3}{8}$ in.; gear A, on spindle, has 20 teeth and gear B, on stud, has 80 teeth. Give answer in tabular form using no gear of less than 24 teeth. Use as few different gears as possible.

75. In a screw cutting train, Fig. 147, page 165, find the proper numbers of teeth for the change gears to cut threads per inch as follows: 6, 8, 9, 11, $11\frac{1}{2}$, 12 and 13. Lead screw has a lead of $\frac{5}{16}$ in., gear A on spindle has 24 teeth and gear B on stud 75 teeth. Give answer in tabular form, no gear to have less than 18 teeth. Use as few different gears as possible.

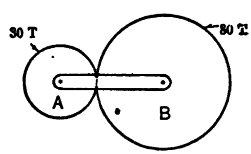
76. Design a train to give a train value of 170, using as few gears as possible, but using no gear of less than 18 teeth or more than 110 teeth.

77. Let the value of a train be exactly 552. Find the least number of pairs necessary and make a sketch showing the arrangement and giving the number of teeth on each gear. No gear to have more than 120 teeth nor less than 20 teeth.

78. Find the number of teeth for a train having a value of 183, with an allowable variation of not more than $\frac{1}{4}$ of 1 per cent. The gears to have not more than 120 nor less than 25 teeth. Use as few pairs of gears as possible.

79. Value of a train is to be approximately 113, with an allowable variation of 1. Find the least number of pairs and sketch the train, indicating the numbers of teeth on each gear. No gear to have more than 50 teeth nor less than 12 teeth.

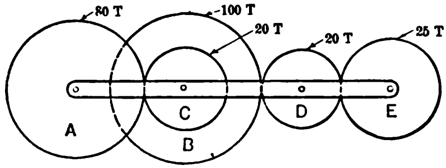
80. In a hoisting train a man exerts a force of 70 lb. at the end of a crank 2 ft. long, and the pitch diameter of the rope drum is 21 in. If 20 per cent. is lost in friction and 9856 lb. are lifted, find the angular velocity ratio of crank to drum and then design a suitable train to connect the two, using the least number of gears possible. Sketch the mechanism showing the arrangement of the gears. No gear to have more than 100 teeth nor less than 18 teeth.



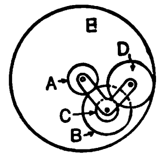
Prob. 81.

81. (*a*) If A is a fixed gear and the arm makes $+3$ turns, find turns of B. (*b*) If A makes $+4$ turns and B makes -11 turns, find turns of arm. (*c*) To what must the number of teeth on A be changed in order that A may make $+15$ turns, B -6 turns, and the arm $+3$ turns?

82. B and C are fast to each other and turn on a fixed axis which is also the axis of the arm. If A makes $+5$ turns and the arm -3 turns, find the turns of B and of E. If D were removed and E placed in contact with B what would be the turns of E?



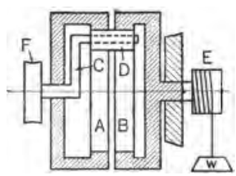
Prob. 82.



Prob. 83.

83. A has 60 teeth, B 80, C 25, D 75, and E 150. (*a*) With A fixed and the arm turned -2 turns, find turns of E. (*b*) With E fixed and A free to turn, find turns of the arm for $+7$ turns of A.

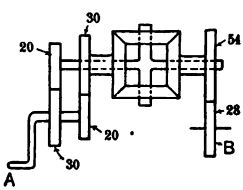
84. Diameter of hoisting drum E is 7 in. Diameter of chain drum F is 10 in. A is a fixed annular having 40 teeth. B is an annular having 42 teeth fast to the hoisting drum E. D is a broad-faced pinion on arm C. How large a weight can be raised by a pull of 40 lb. in the chain, neglecting friction?



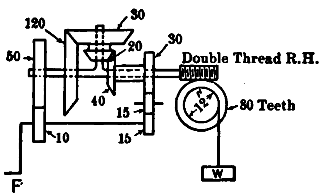
Prob. 84.

85. In the Triplex Pulley-block, Fig. 157, page 180, E has 75 teeth, C 20 teeth, D 160 teeth; diameter of $A=30$ in., and diameter of $G=6$ in. If 60 lb. raise 7500 lb., how many teeth on F? (Neglect friction.)

86. For two turns of the handle A how many turns does B make? Does it turn in the same direction as A or opposite?



Prob. 86.



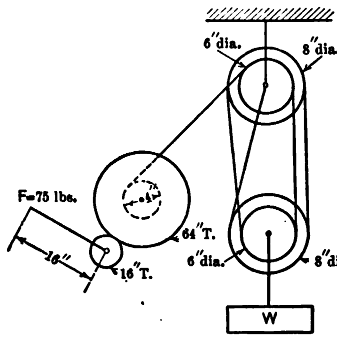
Prob. 87.

87. With numbers of teeth as shown, how many turns and in what direction must the crank F be turned to raise W 6 in.?

88. Find W if there is a friction loss of 40 per cent.

89. A differential pulley-block is to lift 1500 lb. with a pull of 30 lb., friction being neglected. Find the ratio of the larger diameter of the upper sheave to the smaller one.

90. In a differential pulley, the smaller diameter of the upper sheave is 12 in. It is found necessary to haul over 7 ft. of chain to raise the weight 6 in. What is the other diameter of the upper sheave? Neglecting friction, what weight would be raised by a pull of 40 lb.?

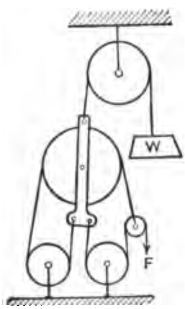


Prob. 88.

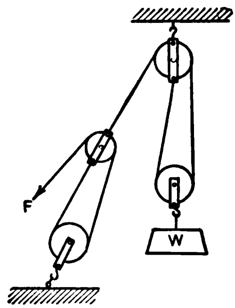
91. With a differential pulley, if the diameters of the sheaves in the fixed block are 12 in. and 11 in., and if the weight of the lower block is 20 lb., what net weight can be raised by a pull of 120 lb. on the chain, allowing a loss of 30 per cent. in friction? How much chain must be overhauled to lift the weight one foot?

92. In this hitch, what force F is required to raise a weight W of 1400 lb., friction being neglected?

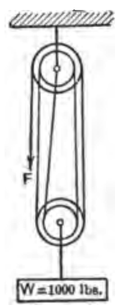
93. If $W=3000$ lb., find the force F.



Prob. 92.



Prob. 93.



Prob. 94.

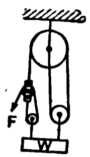
94. Two men, weighing 150 lb. each, stand on W and pull just enough to sustain the load.

(*a*) What pull do they exert on the rope?

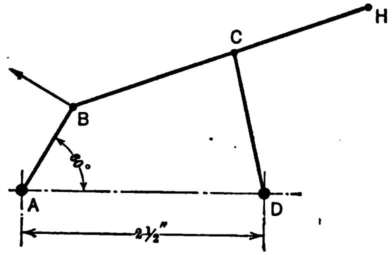
(*b*) What is the tension on the support for the upper block, neglecting the weight of the blocks and rope itself?

(*c*) If the men stood on the ground what would be the tension in the rope which supports the upper block?

95. What force F is necessary to raise a weight W of 192 lb. if 40 per cent. is lost in friction?

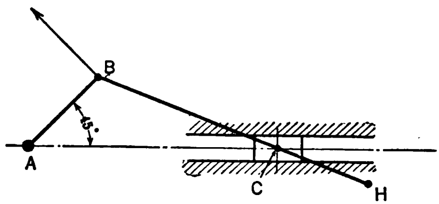


Prob. 95.



Prob. 96.

96. A and D are fixed axes. $AB=1$ in., $BC=1\frac{3}{4}$ in., $BH=3\frac{1}{4}$ in., $DC=1\frac{1}{2}$ in. Find the instantaneous axis of BCH. Assuming the velocity of B to be reprêsented by a line $\frac{3}{4}$ in. long, find the linear velocity of the point H on the rod BCH.



Prob. 97.

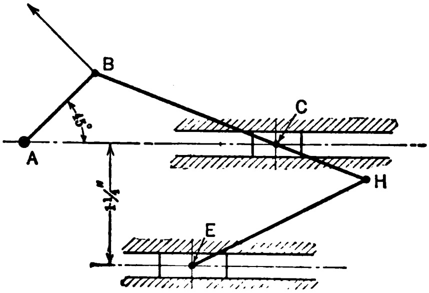
97. A is a fixed axis. $AB=1$ in., $BC=2$ in., $BH=3$ in. Assuming the velocity of B to be represented by a line 1 in. long, find velocity of C and of H.

98. A is a fixed axis. $AB=1$ in., $BC=2$ in., $BH=3$ in., $HE=2$ in. Assuming l.v. of $B=1$ in., find l.v. of E.

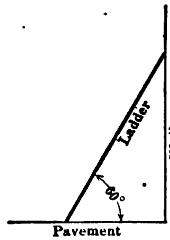
99. The ladder shown is 12 ft. long. If the lower end starts to slip on the pavement at the rate of 1 ft. per second, find graphically the magnitude and direction of the velocity of a point on the ladder 4 ft. from the lower end when in the position shown.

100. A and G are fixed axes. $GF=1\frac{3}{16}$ in., $FE=2\frac{3}{8}$ in., $ED=\frac{7}{8}$ in., $DB=1\frac{3}{8}$ in., $AB=1\frac{3}{4}$ in., $DC=3\frac{1}{8}$ in., $CH=4\frac{3}{4}$ in. Find the instantaneous axes of EF, CD, and CH. Assuming that the l.v. of F is represented by a line 1 in. long, find the l.v. of C and of H.

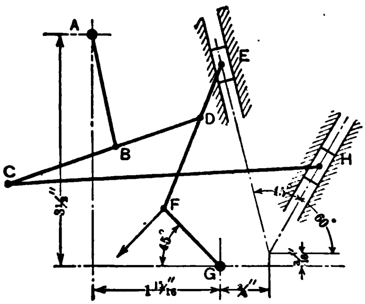
101. B and P are fixed axes. ABC is a rigid link. $AB=1\frac{1}{2}$ in., $BC=1\frac{3}{4}$ in., $CM=1\frac{1}{8}$ in., $CK=2\frac{1}{8}$ in., $KH=3\frac{3}{4}$ in., $KG=2$ in., $PH=$



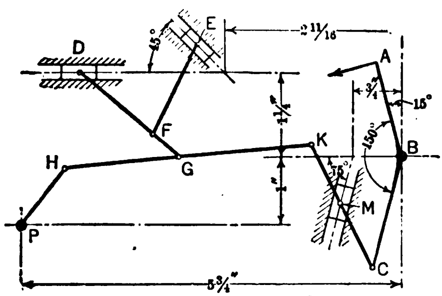
Prob. 98.



Prob. 99.



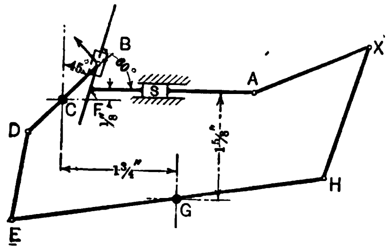
Prob. 100.



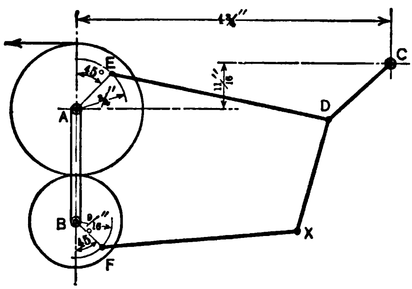
Prob. 101.

$1\frac{1}{16}$ in., $GD=2$ in., $GF=\frac{1}{2}$ in., $FE=1\frac{1}{2}$ in. Assuming a l.v. of $A=\frac{3}{4}$ in., find l.v. of D and of E, showing the instantaneous axes necessary for the solution.

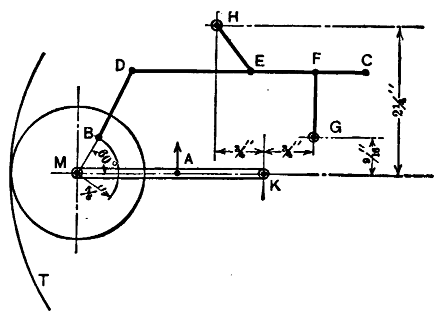
102. C and G are fixed axes. $CB = \frac{3}{4}$ in., $CD = \frac{3}{4}$ in., $DE = 1\frac{3}{8}$ in., $GE = 2\frac{1}{2}$ in., $GH = 2\frac{1}{4}$ in., $HX = 2\frac{1}{8}$ in., $FA = 2\frac{1}{2}$ in., $AX = 1\frac{7}{8}$ in. Assuming a l.v. of the pin at B equal to $\frac{9}{16}$ in., find l.v. of A, H, and X.



Prob. 102.



Prob. 103.



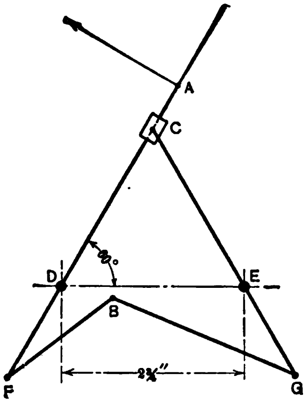
Prob. 104.

103. A is a disc 2 in. diameter and B is a disc $1\frac{3}{8}$ in. diameter, both turning on fixed axes. A drives B with no slipping. C is a

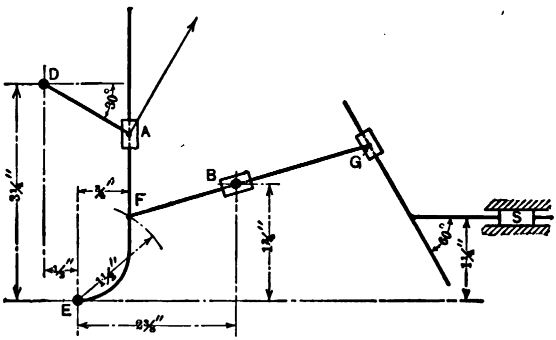
fixed axis. $ED = 3\frac{3}{8}$ in., $CD = 1\frac{1}{4}$ in., $DX = 1\frac{3}{4}$ in., $FX = 3$ in. If the surface speed of A is 1 in., find the l.v. of X.

104. K, G, and H are fixed axes. The wheel is 2 in. diameter and rolls on track T without slipping. $KM = 2\frac{13}{16}$ in., $KA = 1\frac{5}{16}$ in. $BD = 1\frac{1}{8}$ in., $GF = 1$ in., $HE = \frac{7}{8}$ in., $DC = 3\frac{1}{2}$ in., $DF = 2\frac{3}{4}$ in., $DE = 1\frac{13}{16}$ in. If the l.v. of A is $\frac{1}{2}$ in., find l.v. of C.

105. D and E are fixed axes. $DA = 3\frac{1}{2}$ in., $DF = 1\frac{5}{8}$ in., $EC = 2\frac{3}{4}$ in., $EG = 1\frac{9}{16}$ in., $GB = 3$ in., $FB = 2$ in. If the l.v. of A is 2 in., find l.v. of B.



Prob. 105.



Prob. 106.

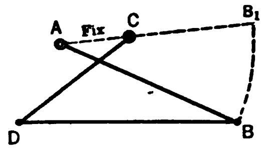
106. Block B is fixed in position, but may oscillate and the rod FG is free to slide through it. Given l.v. of the pin at A equal to 2 in., find l.v. of slide S. $DA = 1\frac{1}{2}$ in., $FG = 3\frac{3}{4}$ in.

107. Plot a curve showing ratio $\frac{\text{a.v. } CD}{\text{a.v. } AB}$ for 30° intervals of AB, starting with AB in position AB_1 and turning uniformly left-handed.

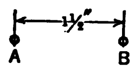
Ordinates = a.v. ratio (1 in. = unity).

Abscissæ = angular positions of AB ($\frac{1}{4}$ in. = 30°).

$AC = \frac{3}{4}$ in., $DC = 1\frac{1}{2}$ in., $DB = 2\frac{1}{4}$ in., $AB = 2$ in.



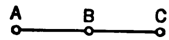
Prob. 107.



Prob. 108.

108. Draw a pantograph to connect the points A and B so that the motion of A shall be to the motion of B as 13 is to 7. Calculate the distance from B to the fixed point. The pantograph is to be so arranged that A may move at least 5 in. to the right along the line through A and B as now situated.

109. Design a pantograph to reduce the motion of the cross-head of an engine that has a stroke of 12 in., to 3 in. so that a steam engine indicator may be operated from it. Let the distance from the fixed point of connection to the cross-head be 15 in. Draw half size.



Prob. 110.

110. The three points A, B, and C are to be connected by a pantograph so that A may move up 4 in., B up 3 in., and C down 2 in. $AC = 6$ in. Locate the fixed point and the point B, and then draw a pantograph that will allow A to be moved 4 in. in every direction.

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